Tentamen Continuummechanica (17-12-2011; docent: Spakman)

Write clearly and provide arguments in derivations and answers. Point scores are given (30 in total).

Useful equations: $\frac{d\rho}{dt} + \rho\nabla.\vec{v} = 0$, $\rho\frac{dv_i}{dt} = \frac{\partial\sigma_{ij}}{\partial x_j} + F_i$, $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathrm{v}_\mathrm{k}\frac{\partial}{\partial \mathrm{x}_\mathrm{k}}$, $\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\theta\delta_{ij}$

$\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\epsilon}_{ij}$, $\rho\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j \partial x_j} + F_i$

1) (5 points; ~17%)

Derive the constitutive equation of an isotropic linear elastic medium from the following two equations: $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$ and $C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

2) (5 points; ~17%)

The postulate of linear viscosity states $\sigma'_{ij} = 2\eta\varepsilon'_{ij}$ and relates the deviatoric stress, to viscosity and deviatoric strain rate. Derive from this postulate the constitutive equation $\sigma_{ij} = -p\delta_{ij} + \eta(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ assuming an incompressible fluid (Note: define p).

3) (12 points ; 40%)

Assume that the following stress field exists in a x_1-x_2-x_3 -frame:

$$\bar{\bar{\sigma}} = \begin{bmatrix} x_3 & x_1x_2 & 0 \\ x_1x_2 & x_3 & 0 \\ 0 & 0 & x_3 \end{bmatrix}$$

a) (2) Determine the body force ("volumekracht" in Dutch).
b) (2) Determine the principal stresses ("hoofdspanningen" in Dutch) in P(1,2,1)
c) (3) Determine the principal axes ("hoofdspanningsassen" in Dutch) in P(1,2,1)
d) (2) Determine the traction at P(1,2,1) acting on the plane $4x_1 + 4x_2 = 12$.
e) (3) Determine the normal and shear stress components of the traction determined in d).
(If you could not find the answer to d) then take $\bar{\sigma}^n = 1/\sqrt{2}[3,3,0]^T$)

4) (8 points ; 26%)

Consider a two-dimensional infinite laminar (parallel) linearly viscous flow in x_1 direction. In depth direction the flow is confined between the depths $x_2=0$ and $x_2=h$.
The viscosity η is constant.

The flow velocity field is given as $\vec{v}(x_1, x_2) = \begin{pmatrix} \frac{\alpha}{\eta}x_2 + x_2 \\ 0 \end{pmatrix}$. The pressure in the fluid is given by $p(x_1, x_2) = \rho g x_2 + \alpha x_1$ and the body force is given by $F_1 = 0, F_2 = \rho g$

a) (2) Determine the strain-rate tensor at (x_1, x_2)
b) (2) Determine the stress tensor at (x_1, x_2)
c) (4) Show that the flow field is a solution of the Navier-Stokes equation.