Examination Paper: GEO3-1302

Continuum mechanics and rheology of the crust and mantle
PART II (SPIERS)

Special Paper for Lisanne Douma
Friday 21-12-2012, 14.00-17.00, HPT Area

N.B.
- The exam paper consists of 5 questions. **Answer 4 of the 5 questions.**
- Take about 45 minutes to answer each question; each question carries equal points
- Answer in English or in Dutch
- Identify all mathematical symbols you use

Good Luck, Lisanne!!!

Question 1

a) Describe the essential nature of the elastic behaviour of dry rock, illustrating your answer with sketches of i) stress-strain and ii) strain-time diagrams.

b) The elastic response of isotropic, porous rock containing a pore fluid at pressure P_f is given by the "poro-elastic" equation

$$2G\,\varepsilon_{ij} = \sigma_{ij} - \frac{1}{3}(1 - 2G/3K)\,\sigma_{kk}\,\delta_{ij} + \frac{2}{3}G(1/K_g - 1/K)P_f\,\delta_{ij}$$

where σ_{ij} and ε_{ij} represent the states of stress and strain respectively. Define fully the quantities G, K, K_g and δ_{ij}.

c) A volume of isotropic porous sandstone, within a gas reservoir at a depth of about 3 km, experiences an in-situ state of compressive stress defined by the tensor

$$\sigma_{ij} = \begin{bmatrix} 70 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{bmatrix} MPa$$

and a pore fluid pressure of 35 MPa. Note that $\sigma_{11} = \sigma_1 = 70$MPa is the vertical principal stress. Assuming that σ_{ij} does not change, use the poro-elastic equation to calculate the state of strain in the rock, relative to the fully unstressed condition. Take $G = 7$ GPa, $K = 10$ GPa and $K_g = 35$ GPa. Calculate also the change in the state of strain if the pore fluid pressure is dropped to 10 MPa as a result of gas production.

d) In the field case, will the state of strain change in the same way when the pore fluid pressure is dropped during gas production? Explain your answer.

Question 2

a) Write down the Coulomb criterion for shear failure of dry isotropic rock and an expression giving the orientation of the failure plane normal. (**N.B. identify all terms appearing and indicate any restrictions on the orientation of the failure plane normal).**

b) Show how the Coulomb criterion is modified when pore fluid is present at a pressure P_f and explain what is meant by the law of effective stress for shear failure.

c) Seismic events in an area of regional tectonic compression indicate reactivation of thrust faults at a depth of h ≈ 4000 m. Assuming (i) that the pore pressure at this depth takes a value equal to 50% of the lithostatic pressure (ρgh), (ii) that σ_3 is vertical and equal to the lithostatic pressure (ρgh), and (iii) that the reactivated faults are healed and characterized by a cohesive shear strength of ~ 2 MPa and a coefficient of internal friction of ~ 3/4, obtain an estimate of the value of the horizontal stress (σ_1) associated with faulting. Take the density (ρ) of the overburden to be 2500 kg/m^3 and g = 10 ms^{-2}.

Question 3

a) Draw a simple 3-D diagram illustrating the key features of an edge dislocation.

b) Explain what is meant by the terms
   i) dislocation glide or slip
   ii) slip system.
   iii) dislocation climb

c) In an unstressed crystal, a straight segment of edge dislocation is pinned between two obstacles, separated by a distance L, lying in the dislocation slip plane. In the region between the obstacles, the dislocation can glide in its slip plane. Explain how this configuration can act as a so-called Frank-Read source of dislocations (use diagrams).

d) Write down an equation for the critical shear stress required to activate a Frank-Read source, stating what equilibrium condition this equation represents.

e) Use the above equation and background knowledge obtained from the course to estimate the yield stress of a crystal containing pinned dislocation segments of length L = 10 μm.

Question 4

a) Explain what is meant by the term steady state creep, illustrating your answer with sketches of stress-strain and strain-time diagrams.

b) List the main mechanisms by which steady-state creep can occur in crystalline materials, indicating the essential nature of each mechanism.

c) Go on to explain briefly the concept of the deformation mechanism map, illustrating your answer with a schematic labelled diagram.

d) What is the physical meaning of the fields and field boundaries in such a map?

Question 5

a) Sketch a typical strength profile for extension/rifting of the continental lithosphere. Assume that extension of the lithosphere occurs by means of symmetric local stretching and thinning. In your sketch, make sure you indicate the important compositional layers in the crust and upper mantle.

b) Go on to list the steps that you would take to turn your sketches into a quantitative rheological model describing lithospheric strength, assuming a fixed extensional strain rate commensurate with plate rifting rates.