

**Examination Paper: GEO3-1302**

**Continuum mechanics and rheology of the crust and mantle  
PART II (SPIERS)**

30-01-2015, 13.30-16.30, Educ-Beta

Note: an extra half hour is available to students who have registered special needs in advance

- N.B.**
- This exam paper consists of 5 questions. **Answer 4 of the 5 questions.**
  - Take about 45 minutes to answer each question; each question carries equal points
  - Answer in English or in Dutch
  - Identify all mathematical symbols you use
  - If you do not understand the English used in a question, raise your hand for help

**Tip:** - Read the questions carefully and answer what is asked. Check answers before leaving!  
**Good Luck!!!**

**Question 1**

- a) Describe the essential characteristics of the elastic behaviour of dry rock, illustrating your answer with sketches of i) stress-strain and ii) strain-time diagrams.
- b) The elastic response of isotropic, porous rock containing a pore fluid at pressure  $P_f$  is given by the "poro-elastic" equation

$$2G \varepsilon_{ij} = \sigma_{ij} - \frac{1}{3} (1 - 2G/3K) \sigma_{kk} \delta_{ij} + \frac{2}{3} G (1/K_g - 1/K) P_f \delta_{ij}$$

where  $\sigma_{ij}$  and  $\varepsilon_{ij}$  represent the states of stress and strain respectively. Define the quantities  $G$ ,  $K$ ,  $K_g$ ,  $\delta_{ij}$  and  $\sigma_{kk}$  fully, using diagrams to illustrate the first three.

- c) An isotropic reservoir sandstone, at 3 km depth in a new gasfield, experiences a state of total compressive stress defined by the tensor

$K = \frac{\sigma_{11}}{2\varepsilon_{12}}$

$$\sigma_{ij} = \begin{pmatrix} 65 & 0 & 0 \\ 0 & 50 & 0 \\ 0 & 0 & 50 \end{pmatrix} \text{ MPa}$$

$\varepsilon_1 = \frac{1}{E} (\frac{\sigma_1}{1} - \frac{\sigma_2}{1} - \frac{\sigma_3}{1})$

where  $\sigma_{11}$  is the vertical stress and  $\sigma_{22} = \sigma_{33}$  are the horizontal stresses. The gas pressure in the reservoir before production is 35 MPa. Assuming that the stress state remains constant, use the poro-elastic equation to calculate the change in the state of strain in the reservoir if the pore fluid pressure is reduced by gas production to 5 MPa.

- d) If the reservoir is 200 m thick estimate the displacement of the reservoir top that would accompany the above reduction in gas pressure.
- e) Use your results to estimate what would happen at the surface when gas is produced, and evaluate whether the assumption of constant stress in the reservoir is reasonable or not.

**Question 2**

- a) Explain what is meant by the term "failure criterion" for brittle rock.
- b) Write down criteria for the two main modes of brittle failure that can occur in regions of the Earth's crust where pore fluid is present at a pressure  $P_f$ . Make sure you identify all terms appearing and indicate any restrictions on the orientation of the failure planes involved.

mathematical relation

- extensional fracture with  $(\sigma_1) = \tau$

- Coulombs criterion

↳ wet  $\tau = \tau_0 + \mu(\sigma_n - P_f)$

- low  $\sigma$ , low  $\varepsilon$

- c) Seismic events in an area of regional tectonic compression indicate reactivation of healed thrust faults at a depth of  $h \approx 6000$  m. Assuming (i) that the pore pressure at this depth takes a value equal to 50% of the lithostatic pressure ( $\rho gh$ ), (ii) that  $\sigma_3$  is near-vertical and equal to the lithostatic pressure ( $\rho gh$ ), and (iii) that the local fault rocks are characterized by a (healed) cohesive shear strength of 10 MPa and a coefficient of internal friction of 6/10, obtain an estimate of the value of the horizontal stress ( $\sigma_1$ ) and the differential stress associated with faulting. Take the overburden density ( $\rho$ ) to be  $2500 \text{ kg/m}^3$  and  $g = 10 \text{ ms}^{-2}$ .  $(\sigma_1 - \sigma_3)$
- d) If the region was one of extension not compression, would earthquakes sourced at 6 km be associated with a smaller or larger differential stress and would they be smaller or bigger?

**Question 3**

- a) Draw simple diagrams illustrating the main features of an edge dislocation.  $w \approx \mu b^2$
- b) Define the terms "dislocation self energy" and "line tension". Illustrate your answer with appropriate formulas, identifying all terms.  $\frac{w \approx \mu b^2}{V}$
- c) Explain what is meant by the term "slip system" and write down an expression for the force (per unit length) on a dislocation lying within its slip system in a stressed crystal.  $F_{xs} = F$   
with  $\tau = 0$   
?
- d) Explain the operation of the Frank-Read source of dislocations. Make sure you mention the various competing forces involved and write down the equilibrium condition required to activate such a source.  $\frac{1}{2}$  circle  
 $R_c = \frac{1}{2} L ? \mu b ?$
- e) Use the above equation and background knowledge obtained from the course to estimate the yield stress of a crystal containing pinned dislocation segments of length  $L = 0.5 \mu\text{m}$ .

**Question 4**

- a) Explain what is meant by the term steady state creep, illustrating your answer with sketches of stress-strain and strain-time diagrams.
- b) List the main mechanisms by which steady-state creep can occur in crystalline materials, indicating the essential characteristics of each mechanism.
- c) Go on to explain briefly the concept of the deformation mechanism map, illustrating your answer with a schematic labelled diagram.
- d) How is such a map constructed and what is the significance of the field boundaries?
- e) How would you use a deformation mechanism map to estimate the flow behaviour of a calcite mylonite containing specific metamorphic minerals defining its metamorphic grade, assuming deformation at geological strain rates?

**Question 5**

- a) Write down and explain what is meant by Byerlee's rule.  $\tau = \mu (\sigma_n - Pf)$
- b) Indicate what deformation mechanisms and what type of constitutive equations are usually used to describe the ductile flow of quartz in the mid-lower crust and of olivine in the upper mantle.  $\dot{\epsilon} \propto \dot{\gamma}$   
flow
- c) Given Byerlee's law and suitable laboratory equations for the ductile flow behaviour of quartz and olivine, list the steps that you would take to construct a strength profile for a section of continental lithosphere undergoing rifting, assuming a uniform extensional strain rate of say  $3 \times 10^{-15} \text{ s}^{-1}$ ?
- d) Finally, list and explain the main problems or weaknesses that you see in the classical approach to constructing a strength profile for a portion of lithosphere  $\dot{\epsilon} \propto \dot{\gamma}$   
- localized shear  
- purely quartz and olivine  
- pure fluid processes

**Beer Award Borrel:** All members of the class are invited for the beer award ceremony and borrel on Thursday 7 April, 17.00. Location follows via e-mail. **CHRIS**

$$\dot{\epsilon} = \bar{A} \exp(-\Delta H/RT) \cdot (\sigma_1 - \sigma_3)^N$$

$$\dot{\gamma} = A' \exp(-\Delta H/RT) \frac{\dot{\epsilon}^N}{\tau}$$