DIVA: Mid-term test 1a - 2015

Write your name and write READABLE ! Each assignment is 1 point

1. Find the limit of the sequence $\lim_{n\to\infty} \frac{(n+1)^2}{\sqrt{3+5n^2+6n^4}}$

2. For the series $\sum_{n=1}^{\infty} e^{-n\ln 3}$ determine the sequences a_n , $S_n = \frac{a(1-r^n)}{1-r}$ and $R_n = S - S_n$ and their limits for $n \to \infty$

3. Determine if the following series converges: $\sum_{n=1}^{\infty} \frac{100^n}{n^{200}}$

4. Find the convergence interval for the series $\sum_{n=1}^{\infty} (-1)^n 2^n (\sin x)^n$

5. If the Taylor series for $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ find the Taylor series for $\frac{\sin\sqrt{x}}{\sqrt{x}}$

6. a. Write in the form $a+bi$: $\frac{1}{z-i}$ if $z = 2-3i$

   b. Determine the absolute value of $\left(\frac{1+i}{1-i}\right)^5$

7. Solve the complex equation for *x* and *y*: $\frac{x+iy}{x-iy} = -i$

8. Test the complex series $\sum_{n=0}^{\infty} \left(\frac{1+i}{2-i}\right)^n$ for convergence

9. Find the convergence circle for the complex series $\sum_{n=1}^{\infty} \frac{(-1)^n z^{2n}}{(2n)!}$

10. a) Write in the form of $x + iy$: $\left(\frac{1-i}{\sqrt{2}}\right)^{40}$

    b) Find the complex roots of $\sqrt[3]{-8i}$

1 $\boxed{1.2.2}$

$$\frac{(n+1)^2}{\sqrt{3+5n^2+6n^4}}$$ hoogste machten: $$\frac{n^2}{n^2\sqrt{6}}$$

dus limiet $= \frac{1}{\sqrt{6}}$

2 $\boxed{1.4.4}$

$$\sum_1^\infty e^{-n\ln 3} = \sum_1^\infty e^{\ln\frac{1}{3}} = \sum_1^\infty \frac{1}{3^n}$$

$$a_n = \left\{\frac{1}{3^n}\right\}_{n=1}^\infty \qquad S_n = \frac{a(1-r^n)}{1-r}$$

$$= \frac{\frac{1}{3}\left(1-\frac{1}{3^n}\right)}{1-\frac{1}{3}} = \frac{1}{2}\left(1-\frac{1}{3^n}\right)$$

$$\lim a_n = 0 \qquad \lim S_n = \frac{1}{2}(1-0) = \frac{1}{2} = S$$

$$R_n = S - S_n = \frac{1}{2} - \frac{1}{2}\left(1-\frac{1}{3^n}\right) = \cancel{\frac{1}{2}} - \cancel{\frac{1}{2}} + \frac{1}{2.3^n}$$

$$R = \lim R_n = \lim \frac{1}{2.3^n} = 0$$

3 $\boxed{1.6.27}$ $$\sum_0^\infty \frac{100^n}{n^{200}}$$

$$\rho_n = \left|\frac{100^{n+1}}{(n+1)^{200}} \cdot \frac{n^{200}}{100^n}\right| \rightarrow \rho = \lim \rho_n = \left|100 \cdot \frac{n^{200}}{n^{200}}\right| = 100$$

hence . divergent

4 $\boxed{1.10.25}$ $$\sum_{0}^{\infty} (-1)^n \cdot 2^n \cdot (\sin x)^n$$

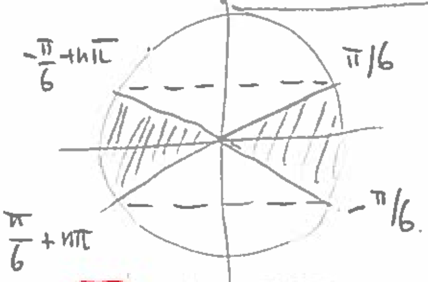
$$\rho = \lim \left| \frac{2^{n+1}}{2^n} \cdot \frac{(\sin x)^{n+1}}{(\sin x)^n} \right| = |2 \sin x| < 1$$

$$\rightarrow |\sin x| < \frac{1}{2} \rightarrow n\pi - \frac{\pi}{6} < x < n\pi + \frac{\pi}{6}$$

Grenzen: $\boxed{x = -\frac{\pi}{6} \rightarrow \sin x = -\frac{1}{2}}$

$$\rightarrow \sum_{0}^{\infty} (-1)^n 2^n \left(-\frac{1}{2}\right)^n = \sum 1 \rightarrow \underline{\text{div}}$$

$\boxed{x = +\frac{\pi}{6} \rightarrow \sin x = +\frac{1}{2}}$



$$\rightarrow \sum_{0}^{\infty} (-1)^n 2^n \left(+\frac{1}{2}\right)^n = \sum_{0}^{\infty} (-1)^n \rightarrow \underline{\underline{\text{div}}}$$

(alternerende reeks test faalt)

5 $\boxed{1.13.11}$ $$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$$

$$\rightarrow \frac{\sin \sqrt{x}}{\sqrt{x}} = \left(\sqrt{x} - \frac{(\sqrt{x})^3}{3!} + \frac{(\sqrt{x})^5}{5!} - \frac{(\sqrt{x})^7}{7!} \right) / \sqrt{x}$$

$$= 1 - \frac{(\sqrt{x})^2}{3!} + \frac{(\sqrt{x})^4}{5!} - \frac{(\sqrt{x})^6}{7!}$$

$$= 1 - \frac{x}{3!} + \frac{x^2}{5!} - \frac{x^3}{7!} + \dots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \cdot x^n}{(2n+1)!}$$

6 $\boxed{2.5.22}$ a

$$\frac{1}{z-i} \quad \text{if} \quad z = 2-3i$$

$$\rightarrow \frac{1}{z-i} = \frac{1}{2-3i-i} = \frac{1}{2-4i} \times \frac{2+4i}{2+4i} = \frac{2+4i}{4+16} = \frac{1}{10}+\frac{2}{10}i$$

$\boxed{2.5.34}$ b absolute value of $\left(\frac{1+i}{1-i}\right)^5 = \left(\frac{\sqrt{2}}{\sqrt{2}}\right)^5 = 1^5 = 1$

7 $\boxed{2.5.46}$

$$\frac{x+iy}{x-iy} = -i \rightarrow x+iy = -i(x-iy) = -ix-y$$

OF: $\frac{x+iy}{x-iy} = -i \rightarrow \frac{x+iy}{x-iy} \cdot \frac{x+iy}{x+iy} = -i$

$$\rightarrow \frac{x^2+2ixy-y^2}{x^2+y^2} = -i \rightarrow x^2-y^2+2ixy = -i(x^2+y^2)$$

So: $x^2-y^2=0$ and $2xy = -(x^2+y^2)$

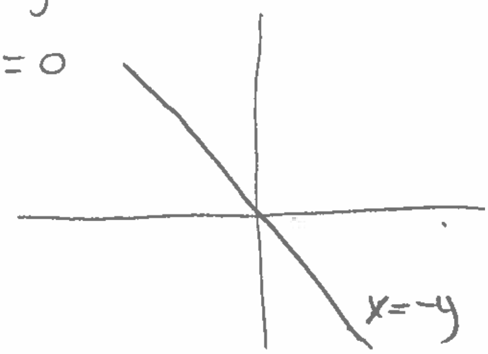
$x^2=y^2$. $\rightarrow x^2+2xy+y^2=0$

$(x+y)^2=0$

$\cancel{x=y} \vee x=-y$

$\rightarrow x=-y$

hence: $x=-y$



8 $\boxed{2.6.13}$ $\sum \left(\frac{1+i}{2-i}\right)^n$

$$C = \lim \left| \left(\frac{1+i}{2-i}\right)^{n+1} \cdot \left(\frac{2-i}{1+i}\right)^n \right| = \left| \frac{1+i}{2-i} \right| = \frac{\sqrt{2}}{\sqrt{5}} < 1$$

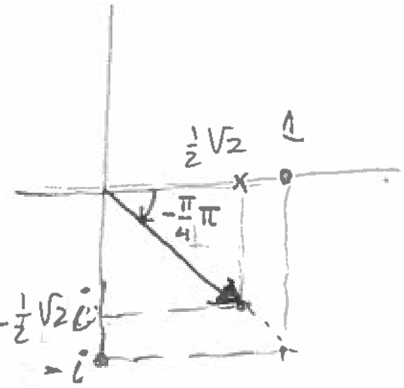
hence convergent.

9 $\boxed{2.7.7}$ $\sum \frac{(-1)^n z^{2n}}{(2n)!}$

$$\rho = \lim \rho_n = \lim \left| \frac{z^{2(n+1)}}{(2(n+1))!} \cdot \frac{(2n)!}{z^{2n}} \right|$$

$$= \lim \left| \frac{z^2}{(2n+2)(2n+1)} \right| = 0 \Rightarrow \underline{\text{for all } z}$$

10 $\boxed{2.9.21}$ a $\left(\frac{1-i}{\sqrt{2}}\right)^{40}$



Hence: $r = \sqrt{\left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2} = 1$

$\vartheta = -\frac{1}{4}\pi$

$$\rightarrow \left(\frac{1-i}{\sqrt{2}}\right)^{40} = \left(e^{-\frac{1}{4}\pi i}\right)^{40} = e^{-10\pi i} = 1$$

$\boxed{2.10.20}$ b $z = \sqrt[3]{-8i} = \sqrt[3]{2^3 \cdot -i}$

since $-i = e^{\frac{3}{2}\pi i} \longrightarrow z = 2e^{\frac{1}{3}\left(\frac{3}{2}\pi + 2n\pi\right)i}$

$n=0 \quad z_1 = 2e^{\frac{1}{2}\pi i} = 2i$

$n=1 \quad z_2 = 2e^{\left(\frac{1}{2}\pi + \frac{2}{3}\pi\right)i} = -\sqrt{3} - i$

$n=2 \quad z_3 = 2e^{\left(\frac{1}{2}\pi + \frac{4}{3}\pi\right)i} = +\sqrt{3} - i$