GEO4-1415 Data processing and inverse theory

Tentamen - 10 Nov 2016 - 13h30-16h00 - BBG-165

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The numbers in () indicate percentage marks for evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch or English.

1. (30) The Discrete Fourier Transform (DFT) is given by the expression

$$F_p = \sum_{n=0}^{N-1} f_n w^{-np} \qquad (1)$$

with $w = exp^{i2\pi/N}$ and $p = (0, 1, ..., N-1)$

We give a causal wavelet $a_t = (2, 0, 2, 0, 2, 0)$. Calculate the zero-frequency sample of the DFT using equation (1). How is F_0 related to the average of a_t? It is obvious that a_t is a sampled version of the function $1 + \cos(2\pi\nu t)$. We want to design a notch filter to remove the average (1 in this case) of this function. The transfer function of a notch filter in the Z-domain is given by:

$$H(z) = \frac{(z - z_0)(z - z_0^*)}{(z - z_p)(z - z_p^*)}, \qquad (2)$$

where $z_0 = \exp[-i2\pi\nu_0\tau]$ and $z_p = (1+\epsilon)\exp[-i2\pi\nu_0\tau]$, ν_0 the frequency which has to be removed and ϵ an arbitrary small number.

Give the time domain expression of the filter and apply this recursive filter to a_t using $\epsilon = 0.1$ and $\epsilon = 0.3$. How could you find an optimal ϵ? What happens if you choose $\epsilon = 0$ or $\epsilon < 0$?

2. (20) We give 2 causal wavelets $a_t = (0, 1, 2, 3)$ and $b_t = (1, 1, 1, 1, 1)$. b_t is the derivative wavelet of a_t. We want to design an optimal derivative filter of length 2, which turns a_t into b_t. Give the coefficients of this optimal filter and apply it to a_t. Does it work? Why do you think your optimal filter is far from the proper derivative wavelet $(1, -1)$?

3. (50) Father Christmas is getting ready for his big day and is sorting his presents out. Amongst many other things, he wants to give all children oranges in their stockings. Of course he wants to be fair and give everybody the same size oranges. He therefore needs to check his supply of oranges. He randomly selects a few oranges of his stock and weighs them. He finds the following:

$$m_1 + m_2 = 2 \qquad (3)$$
$$m_2 + m_3 = 2 \qquad (4)$$
$$m_1 + 2m_2 + m_3 = 4 \qquad (5)$$

Use a singular value decomposition to determine the weights of the 3 oranges. Could you have solved this problem using a least-squares approach or a minimum norm approach? Explain. Would any other method than singular value decomposition given the same answer? Explain. Has father Christmas a good stock of oranges this year?

Good luck.