**Mid-Term "Geodynamics", March 1st, 2017**

(Ex1: 2pts, Ex2: 3pts, Ex3: 2pts, Ex4: 3pts)

# Exercise 1

Derive the following expression for the moment of inertia of a spherically symmetric Earth model with outer radius $R$

$$I = \frac{8\pi}{3}\int_0^R \rho(r) r^4 dr$$

Hint: use the symmetry and compute $I = \frac{1}{3}(I_x + I_y + I_z)$.

# Exercise 2

The density profile of a spherically symmetric planet is given by

$$\rho(r) = \begin{cases} \rho_0 & r \leq R/2 \\ \xi\left(\frac{1}{r} - \frac{1}{R}\right)\rho_0 & r \geq R/2 \end{cases}$$

1. Determine $\xi$ so that the density is continuous at $r = R/2$ and sketch $\rho(r)$.
2. Compute the average density $<\rho>= \frac{1}{V}\int \rho dV$ of the planet as a function of $R, \rho_0$
3. Compute the momentum of inertia $I$ for this density distribution
4. Determine whether $I$ can be written as $I = fMR^2$ where $M$ is the mass of the body.

# Exercise 3

Here are a few equations:

$$\begin{aligned} g &= \mathcal{G}\rho r^2 && (1)\\ F &= \rho g h && (2)\\ x &= \frac{1}{2}at^2 + v_0 t + \sqrt{x_0} && (3)\\ m &= 4\pi\rho^2 r^2 && (4) \end{aligned}$$

where $g$ is the gravity acceleration, $\mathcal{G}$ is the gravitational constant, $\rho$ is the mass density, $r$ is a radial distance, $F$ is a force, $h$ is a height, $x, x_0$ are distances, $a$ is an acceleration, $t$ is the time, $v_0$ is a velocity and $m$ is mass.

1. write down the dimension of each of these parameters in the form of $M^\alpha L^\beta T^\gamma$ where $M$ stands for mass, $L$ for length and $T$ for time.
2. establish for each equation above whether they are plausible by doing a simple dimensional analysis.

# Exercise ~~3~~ 4

Let us consider a three-layer planet composed of a core, a lower-mantle and an upper-mantle with densities $\rho_c$, $\rho_{lm}$, and $\rho_{um}$ respectively. The core-mantle boundary is at $r = R_c$ and the transition between upper and lower mantle occurs at $r = R_m$. The radius of the planet is $R$. In what follows we assume that $\rho_c$, $M$, $R_c$, $R_m$ and $I$ are known.

1. Compute the mass $M_c$ of the core and its moment of inertia $I_c$.
2. Compute the total mass $M$ of the planet and its total moment of inertia $I$, both as a function of $R_c, R_m, \rho_c, \rho_{lm}, \rho_{um}$.
3. Establish a relationship of the form
$$\begin{pmatrix} M - M_c \\ I - I_c \end{pmatrix} = A \cdot \begin{pmatrix} \rho_{lm} \\ \rho_{um} \end{pmatrix}$$
and write explicitely the $A$ matrix.
4. Determine $\rho_{lm}$ and $\rho_{um}$ as a function of $M, M_c, I, I_c, R_c, R_m, \rho_c$