Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

Part II (Spiers) 15-04-2015 13.30 -16.30 hours Room: EDUC GAMMA

- The duration of this exam is 3 hours.
- Answer any **4** of the 7 questions given.
- All questions count with equal weight to the final grade.
- Allow about 45 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you identify all mathematical symbols used in answering the questions
- Use SI units unless otherwise specified.
- If you do not understand the English in any of the questions, raise your hand for help.
  Good luck !!!

Question 1

a) Write down the stress-strain relations for an anisotropic elastic material using **matrix** notation.

b) Taking into account the *symmetry* of the stiffness matrix (C_{rs}), the non-zero components of this matrix for an ***olivine*** crystal (orthorhombic) are specified as follows:-

$$\left.\begin{array}{lll} C_{11} = 3.00 & C_{22} = 1.80 & C_{33} = 2.10 \\ C_{23} = 0.69 & C_{31} = 0.61 & C_{12} = 0.60 \\ C_{44} = 0.58 & C_{55} = 0.71 & C_{66} = 0.70 \end{array}\right\} \times 10^{11}\ \text{Pa}$$

referred to the orthorhombic crystal axes x_1, x_2, x_3. ***Write out the matrix C_{rs} in full.***

c) Explain the physical meaning of the first vertical column of the matrix C_{rs}.

d) An olivine single crystal is subjected to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{pmatrix} 2 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 1 \end{pmatrix} \times 10^{-4} \quad (\text{referred to } x_1, x_2, x_3)$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

e) Calculate also the mechanical work done on the olivine crystal when subjected to the above strain, making sure you state the units!!

f) Use the first and second laws of thermodynamics to **show** how this mechanical work changes the thermodynamic state of the crystal, if deformation occurs at constant temperature, and explain what would happen to the crystal if it is placed in contact with an unstrained olivine crystal with a silicate melt layer between the two crystals.

Question 2

a) Explain what vacancies are and write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T (K) and hydrostatic pressure P. Define all symbols appearing !!

b) Explain the statistical meaning of your answer to (a), with reference to the Boltzmann distribution law.

c) Derive an equation to show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n (superimposed on the hydrostatic component P), and hence explain the theoretical basis (driving force) for solid state diffusion creep.

d) Explain the most likely pathway for vacancy and atomic diffusion during deformation by solid state diffusion creep at temperatures of around 0.7 of the melting point in K. Then identify the physically possible rate limiting steps along this pathway, using an electrical circuit analogue diagram. Which of these steps do you expect to control deformation rate in a typical crystalline solid at the specified temperatures?

e) Write down the theoretical rate equations for ONE of the two best-known processes of solid state diffusion creep and name the process you choose and the associated diffusion path. Go on to obtain an expression for the viscosity of a solid deforming by this mechanism. Finally, identify which quantities in this expression will determine whether the process becomes more or less important with increasing depth in the lower crust or upper mantle.

Question 3

a) Explain what is meant by the term "superplasticity" (i.e. structural superplasticity).

b) Explain what microphysical mechanisms give rise to superplastic flow in metals, ceramics and rocks.

c) What ductile deformation mechanisms do not give rise to superplastic flow and why?

d) Go on to explain how you would determine if the processes responsible for superplastic flow behaviour operated in a fault zone rock found in a field study of a high grade shear zone.

e) Explain **three** ways in which the processes responsible for superplastic flow of rock can be initiated in shear zones and outline the implications for the localization behaviour of major shear zones.

Question 4

a) Draw a sketch of a planar segment of grain boundary penetrated by a solution phase in island-channel or thin film form. If the segment transmits a normal stress σ_n, write down an approximate expression for the average chemical potential of the solid within the grain boundary segment. Identify all of the symbols used and state their units.

b) Consider a granular pack of quartz grains (diameter d) in which the pores are filled with a saturated solution phase. The grain pack is contained in a vertical steel cylinder. If the grain pack is instantaneously subjected to a vertical stress σ and the liquid to a lower hydrostatic pressure p, describe how the system will respond in the short and longer term? Use diagrams.

N.B. Assume i) that interfacial energy driving forces are negligible, ii) that grain contacts are penetrated by fluid in island-channel form, and iii) that deformation within the grains is elastic.

c) Explain what processes might control the rate at which the system would respond in the long term, and which ones you would expect to be slowest under upper crustal conditions.

d) Go on to list the steps you would take to construct a model of the long term response of a gas reservoir sandstone to pore pressure reduction, assuming the reservoir to be located at say 4 km depth (do not derive models).

e) Name at least one other important crustal rock system to which your model might apply.

Question 5

a) Explain, in terms of dislocation dynamics, what is meant by the terms "work hardening" and "recovery".

b) Go on to describe how these processes can lead to steady-state creep in rocks at elevated temperatures.

c) Develop a rate model for deformation by high-temperature, climb-controlled dislocation creep where the obstacles to dislocation glide are attractive junctions (dipoles) between edge dislocations of opposite sign (Weertman type model).

d) This model works well for dislocation creep at high temperatures, but breaks down at low-moderate temperatures. Why is this and how could you improve the model for the lower temperature range?

e) The activation volume for self-diffusion in a dislocation-free portion of crystal lattice is a positive quantity. The activation volume for self-diffusion in dislocation cores is also positive but much smaller. Might this influence the dislocation creep behaviour of olivine at high mantle pressures, and if so how and why??

Question 6

a) Explain what is meant by the term "grain boundary migration" and indicate what thermodynamic forces can drive it.

b) Derive an expression for the velocity of migration (V) of a grain boundary separating two grains in a pure crystalline solid, assuming a free energy difference between the grains of Δf.

c) Explain how and why this expression would be modified if the material contains an aqueous solution phase in grain boundaries (i.e. if grain boundaries are wet).

d) Do you expect the activation energy for grain boundary migration to be higher or lower in wet versus dry materials? Use your answer to sketch a V vs. $1/T$ diagram illustrating how the velocity of migration can be expected to depend on temperature in pure dry and pure wet materials. Then use this plot to indicate the conditions under which you expect fluid -assisted grain boundary migration to dominate over solid state migration.

e) How would you recognise if a natural rock has recrystallized by fluid assisted grain boundary migration as opposed to solid state migration?

Question 7

a) Measured values of the tensile fracture strength (T_0) of brittle materials are usually much lower than the theoretical ("bond strength) value σ_T. Explain why this is.

b) Consider a flat "elliptical" crack (length 2a) within an infinite plate of elastic material (Young's Modulus E) and suppose that this plate (which is of unit thickness) is subjected to a remote uniaxial tensile stress (σ) oriented normal to the crack surface. The applied stress will give rise to a stored elastic energy U_e within the plate. Given that the rate of change of U_e with respect to crack length can be written as

$$\frac{d\,U_e}{da} = \frac{-2\pi a \sigma^2}{E}$$

derive the Griffith failure criterion for uniaxial tensile loading. **State any assumptions made.**

c) Go on to explain why the stress at which a brittle solid breaks, by say Mode I tensile failure, is not a material property.

d) Following on from your answer to Part (c), explain what is meant by the terms "stress intensity factor" and "fracture toughness", and re-write the Griffith criterion in terms of stress intensity factor and critical stress intensity factor.

e) A sample of low porosity but still permeable quartzite is loaded uniaxially, at room temperature and under dry conditions, to 80% of its brittle compressive failure strength. An initial instantaneous shortening of 0.2% occurs when the load is applied, but beyond that no further deformation occurs. The pores of the rock are then flooded with water at atmospheric pressure and the sample immediately fails catastrophically. Give an explanation for this behaviour.