Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

Part II (Spiers) 18-04-2012 13.30 -16.30 hours Room: AW Grote Collegezaal

Note:

- The duration of this exam is 3 hours.
- Answer any **4** of the **7** questions given.
- All questions count with equal weight to the final grade.
- Allow about 45 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you identify all mathematical symbols used in answering the questions (marks will be deducted for unidentified symbols).
- Use SI units unless otherwise specified.

Good luck all !!!

Question 1

a) Write down the stress-strain relations for an anisotropic elastic material
 i) using tensor notation and
 ii) using matrix notation.

b) Taking into account the ***symmetry*** of the stiffness matrix (C_{rs}), the non-zero components of this matrix for an ***olivine*** crystal (orthorhombic) are specified as follows:-

$$\left.\begin{array}{lll} C_{11} = 3.00 & C_{22} = 1.80 & C_{33} = 2.10 \\ C_{23} = 0.69 & C_{31} = 0.61 & C_{12} = 0.60 \\ C_{44} = 0.58 & C_{55} = 0.71 & C_{66} = 0.70 \end{array}\right\} \times 10^{11}\ \text{Pa}$$

referred to the orthorhombic crystal axes x_1, x_2, x_3. ***Write out the matrix C_{rs} in full.***

c) Explain the physical meaning of the vertical columns of the matrix C_{rs}.

d) An olivine single crystal is subjected to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{pmatrix} 4 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \times 10^{-4} \quad \text{(referred to } x_1, x_2, x_3\text{)}$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

e) Calculate also the mechanical work done on the olivine crystal when subjected to the above strain, making sure you state the units!!

f) State how this mechanical work changes the thermodynamic state of the crystal, if deformation occurs at constant temperature (i.e. under isothermal conditions), and explain what would happen to the crystal if it is in contact with with a silicate melt with which it was at equilibrium before the above strain was applied.

Question 2

a) Write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T (K) and hydrostatic pressure P. Define all symbols appearing !!

b) Explain the statistical meaning of your answer to (a), with reference to the Boltzmann distribution law.

c) Derive an equation to show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n (superimposed on the hydrostatic component P), and hence explain the theoretical basis (driving force) for solid state diffusion creep.

$Pa = N/m^2$

$J = Nm$

$J = Pa \cdot m^3$

$Pa = J/m^3$

d) Explain the possible pathways for vacancy and atomic diffusion during deformation by solid state diffusion creep, and identify the physically possible rate controlling steps using an electrical circuit analogue diagram.

e) At high temperatures (say 0.7-0.8 of the melting point in K), which transport path do you expect to be most important for vacancy diffusion in a very fine grained material? Name the corresponding diffusion creep mechanism and write down the theoretical rate equation for this creep mechanism (no need to derive it!). Finally, show how the viscosity of a material deforming by this mechanism is expected to depend on temperature and pressure.

Question 3

a) Explain what is meant by the terms "necking instability" and "superplasticity" (structural), indicating the rheological characteristics of materials showing such behaviour.

b) List the mechanisms of steady state flow that display the kind of rheological behaviour needed for getting necking instability. What is the key factor that they have in common?

c) List the principal microstructural characteristics exhibited by crystalline materials that show "superplastic flow", and explain which deformation mechanisms can lead to such behaviour.

d) Ultramylonitic shear zones developed in both crustal and upper mantle rocks often show evidence for the operation of the processes responsible for superplastic flow. Aside from dynamic recrystallization or cataclasis, explain how such behaviour can become localized in shear zones and outline the implications for the strength of major shear zones compared to surrounding host rock.

e) A high-tech ceramics company is having trouble avoiding cavity growth (cavitation) during production of silicon nitride (Si_3N_4) ceramic components by superplastic forming, despite having addred impurities that produce a small amount of melt at grain boundaries. List 3 ways in which the problem might be solved.

Question 4

a) Explain, in terms of dislocation dynamics, what is meant by the terms "work hardening" and "recovery".

b) Go on to describe how these processes can lead to steady-state creep in rocks at elevated temperatures.

c) Develop a rate model for deformation by high-temperature, climb-controlled dislocation creep where the obstacles to dislocation glide are attractive junctions (dipoles) between edge dislocations of opposite sign (Weertman type model).

d) This model works well for dislocation creep at high temperatures, but breaks down at low-moderate temperatures. Why is this and how could you improve the model for the lower temperature range?

e) Dislocation creep in calcite at high temperatures shows an apparent power law n-value of 7 or 8. Hypothesise whether this suggests climb or cross-slip controlled creep, and explain what experiments you would do to prove or disprove your hypothesis. Give the theoretical basis for your proposed experiments.

Question 5

a) Explain what is meant by dynamic recrystallization and outline the main mechanisms of microstructural change involved in this process.

b) How is the dynamically recrystallized grain size that develops during steady state dislocation-dominated flow typically realted to flow stress and temperature?

c) Recent work has shown that dynamic recrystallization reorganises the grain size of a material to lie in the boundary between dislocation creep (grain size insensitive) and diffusion creep (grain size sensitive). Use this concept to obtain an alternative relationship between steady state flow stress and recrystallized grain size. Hint: Assume that dislocation creep is climb controlled with $3 \leq n \leq 5$ and diffusion creep is dominated by grain boundary diffusion.

d) Is the result you obtain similar to or different from that you obtained in practical classes using the Avrami equation? Why do you think the result is similar or different?

f) Explain under what circumstances dynamic recrystallization can lead to shear zone localization and whether or not recrystallized grain size versus stress relations can then be applied to estimate paleostresses.

Question 6

8 a) Explain the difference between grain boundary diffusional pressure solution and fluid-assisted grain boundary migration. In your answer, address both driving force and kinetic controls.

b) Derive expressions for the velocity of fluid-assisted migration (V) of a grain boundary separating two grains 8 in a crystalline solid, assuming a free energy difference between the grains of Δf (J/mole). Consider all possible rate controlling steps.

? c) What form would Δf take if caused by a difference in dislocation density?

d) By comparing the likely activation energies for grain boundary migration in dry and wet materials (in a V vs. $1/T$ diagram), indicate the conditions under which you expect fluid assisted grain boundary migration to 8 dominate over solid state migration in pure crystalline solids.

e) How would you recognise if a natural rock has recrystallised by fluid assisted grain boundary migration as 8 opposed to solid state migration?

Question 7

a) Measured values of the tensile fracture strength (T_o) of brittle materials are usually much lower than the theoretical ("bond strength) value σ_T. Why is this?

b) Consider a flat "elliptical" crack (length 2a) within an infinite plate of elastic material (Young's Modulus E) and suppose that this plate (which is of unit thickness) is subjected to a remote uniaxial tensile stress (σ) oriented normal to the crack surface. The applied stress will give rise to a stored elastic energy U_e within the plate. Given that the rate of change of U_e with respect to crack length can be written

$$\frac{d\,U_e}{da} = \frac{-2\pi a \sigma^2}{E}$$

derive the Griffith failure criterion for uniaxial tensile loading. **State any assumptions made.**

c) Go on to explain what is meant by the terms
   i) "stress intensity factor" and
   ii) "critical stress intensity factor".
   iii) "Mode I cracks"
and write the Griffith failure criterion in terms of these two quantities (Assume Mode I cracks).

e) Explain what is meant by subcritical crack growth.

f) A sample of sandstone is loaded uniaxially, at room temperature and under dry conditions, to 80% of its brittle compressive failure strength. An initial instantaneous shortening of 0.3% occurs when the load is applied. This is accompanied by a burst of acoustic emission activity. Over a period of 3 weeks thereafter, no time-dependent deformation occurs: the sample length is unchanged and no acoustic emissions are detected. The pores of the rock are then flooded with water at atmospheric pressure. From that moment, acoustic emission activity re-starts and the sample deforms with time. After 1 further week, the sample fails in a brittle manner with no increase in load.

- Draw a strain-time diagram illustrating how the sample behaved as a function of time.
- Consider the instantaneous shortening that occurs upon initial loading: explain what you think causes this strain?
- Offer a mechanistic explanation for each stage of the sample's behaviour after water is added, *taking into account the fact that when water penetrates pre-existing cracks it will instantly reduce the surface energy of the cracks compared to when filled with air!! Think step by step!*