Exam Principles of Groundwater Flow 2016

Friday November 11, 2016 (13:00-17:00)

Success!

1. Consider a fully penetrating pumping well in a confined aquifer. A stationary pumping test is conducted. The well diameter $d_w = 0.4$ m, while the pumping rate is given by $Q = 1000$ m^3/day. The thickness of the aquifer is $D = 20.0$ m. Two observation wells are installed at $r_A = 30.0$ m and at $r_B = 60.0$ m. The observed hydraulic heads are respectively $h(r_A) = 30.0$ m and $h(r_B) = 30.25$ m. The porosity of the aquifer is $n = 0.40$ and the storativity $S = 5 \cdot 10^{-4}$.

   (a) Determine the hydraulic conductivity k [m/day] of the aquifer.

   (b) Show by derivation that the travel time from an arbitrary distance R [m] towards the well screen is given by

   $$t_{travel} = \frac{\pi n D}{Q}(R^2 - r_w^2)$$

   (c) Let's assume that the aforementioned observed hydraulic heads in wells A and B were obtained during a transient pumping test. The initial head distribution h_0 =30.5 m, i.e. at $t = t_0$, the start of the test. Use the well-function graph to determine the time $(t - t_0)$ in days at which the head values in A and B were observed?

   (d) Repeat the previous computation of $(t - t_0)$, but now using the truncated series expansion for the well function $W(u)$.

   (e) Assume we are allowed to use the truncated series expansion, yielding

   $$h_0 - h = \frac{Q}{4\pi k D}\left[-\gamma - \ln(u)\right]$$

   where

   $$u = \frac{Sr^2}{4kD(t - t_0)}$$

Derive an expression for the specific discharge $q(r) = -k\frac{dh}{dr}$. Conclusion?

2. Please indicate whether the following statements are true or false. Motivate your answer!

   (a) In a saturated aquifer the fluid pressure p is always equal to the total vertical stress minus the vertical effective stress.

   (b) According to the differential form of Terzaghi's Law, i.e.

   $$d\sigma_{vt} = dP + d\sigma_{ve}$$

   no subsidence can occur in unconfined aquifers.

   (c) In case of transient flow due to two fully penetrating pumping wells in a confined aquifer, superposition and imaging can be applied to compute the hydraulic head in an arbitrary point (x, y) in the aquifer.

   (d) If the hydraulic head distribution in the vicinity of a pumping well in an unconfined aquifer is given by

   $$h^2(r) = h_w^2 + \frac{Q}{\pi k}\ln(\frac{r}{r_w}),$$

   where r_w is the well radius and h_w the hydraulic head in the pumping well. In that case, the specific discharge at the well screen is given by

   $$q(r_w) = -\frac{Q}{2\pi r_w h_w}$$

3. We consider an excavation site directly next to a straight impermeable subsurface slurry wall. The wall fully penetrates the confined aquifer. Part of the confining top soil layer has to be excavated for building construction purposes. The head in the confined aquifer has to be lowered. If the head in the aquifer is too high, the excavation bottom will heave upward (Dutch: opbarsten). In order to lower the head in the vicinity of the excavation site near the impermeable wall a pumping well is installed at distance b [m] from the wall. The radius of the pumping well is r_w [m] and the pumping rate is Q m^3/s. The thickness of the aquifer is D [m] and the porosity is $n = 0.4$.

(a) Show that the hydraulic head in an arbitrary point (x, y) is given by

$$h(x,y) = \frac{Q}{2\pi kD} \ln\left(\frac{r_1 r_2}{2r_w b}\right) + h_w,$$

where $r_1 = \sqrt{(x-b)^2 + y^2}$ and $r_2 = \sqrt{(x+b)^2 + y^2}$.

(b) If $k = 1.0$ m/day, $b = 50$ m, $r_w = 0.3$ m, Q=30 m^3/hour, $D = 20$ m and the head in the pumping well is $h_w = 30.0$ m, compute the head at the excavation site.

(c) For convenience we rewrite the head distribution as:

$$h(x,y) = \frac{Q}{2\pi kD}[\ln(r_1) + \ln(r_2) - \ln(2r_w b)] + h_w$$

or

$$h(x,y) = \frac{Q}{4\pi kD}[\ln((x-b)^2+y^2)+\ln((x+b)^2+y^2)] - \frac{Q}{2\pi kD}\ln(2r_w b)] + h_w$$

Show that the effective velocity along the positive x-axis is given by

$$v(x,0) = -\frac{Q}{\pi nD}\frac{x}{x^2-b^2}$$

(d) Show by derivation that the travel time along the x-axis from $x = \frac{b}{2}$ towards the well (x=b) is given by

$$t_{travel} = -\frac{\pi nDb^2}{Q}\left[\frac{3}{8} + \ln(\frac{1}{2})\right]$$

(e) Compute the travel time in days.

4. Consider a deep geothermal well. The depth of the well is H [m]. Let's assume that the subsurface consists of homogeneous saturated sand with porosity n [-]. The density of the fluid is ρ_f [kg/m^3] and the density of the solid sediment particles is ρ_s [kg/m^3].

   (a) First we assume that the fluid is incompressible. Determine the fluid pressure at the bottom of the geothermal well.

   (b) Determine the vertical total stress at depth H [m] in the sediment.

(c) Show that the vertical effective stress at depth H [m] equals

$$\sigma_{ve} = (\rho_s - \rho_f)(1 - n)gH$$

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(d) Next we assume that the fluid is compressible. Show that the density of the at depth H [m] is approximately equal to

$$\rho(H) \approx \rho_f(1 + \beta\rho_f gH),$$

where β [1/Pa] is the compressibility of the fluid and g [m/s^2] the acceleration due to gravity. Hint: use the fact that β of the fluid is very small and therefore assume that $\rho \approx \rho_f$.

5. Consider a confined aquifer with thickness D [m] and length L [m]. At the left-hand side of the aquifer a constant head boundary is present: $h(0) = h_0$. At the right-hand side of the aquifer an other constant head boundary is present: $h(L) = h_L$. Assume that $h_0 > h_L$. The hydraulic conductivity is k [m/s]. There is no recharge or discharge.

(a) Determine the hydraulic head distribution.

(b) Let's assume that k has a very high value, implying high velocities in the aquifer. Now Darcy-Forchheimer law has to be used to correctly model the flow in this highly permeable aquifer. Darcy-Forcheimer is given by

$$q(1 + \beta q) = -k\frac{dh}{dx}$$

Show that the specific discharge for this case yields

$$q = -\frac{2k}{1 + \sqrt{1 - 4\beta k\frac{dh}{dx}}}\frac{dh}{dx}$$

(c) Somebody states that the two-dimensional volumetric flow rate Q' AND the specific discharge q must be constant in the aforementioned aquifer, also when Darcy-Forchheimer applies. If that is the case, the only possible hydraulic head distribution is the linear distribution, as found under a). Check wether this is correct or not.

..... The end .....