Principles of Groundwater Flow

1. Consider horizontal stationary flow in an unconfined aquifer with constant recharge N and constant evapotranspiration E. The hydraulic head $h(x)$ satisfies the following ordinary differential equation

$$\frac{d}{dx}\left(K_x h \frac{dh}{dx}\right) + N = E,$$

where K_x denotes the (constant) hydraulic conductivity in the x-direction.

a. If the constant-head boundary conditions are given by $h(0) = h_0$ and $h(L) = h_L$, show that the shape of the water table $h = h(x)$ satisfies

$$\frac{1}{2}(h(x)^2 - h_0^2) = -\frac{(N-E)}{2K_x}(x^2 - xL) + \frac{(h_L^2 - h_0^2)}{2L}x$$

b. Using the definition of the discharge per unit width of the aquifer, i.e.

$$Q' = -K_x\, h \frac{\partial h}{\partial x}$$

show that the corresponding discharge per unit width of the aquifer is given by

$$Q' = K_x \frac{h_0^2 - h_L^2}{2L} - \frac{1}{2}(N - E)(L - 2x)$$

c. Show by derivation that if $h(0) = h(L)$, i.e. equal hydraulic heads at the boundaries, the position of the water-divide is simply given by

$$x_d = \frac{L}{2}$$

d. To avoid salinization of soil in an agricultural area, parallel drainage canals are to be installed to maintain the water table at depth of **at least** 4 m below the ground surface. At that depth, evapotranspiration can be disregarded. This phreatic aquifer has a hydraulic conductivity of 1 m/day. The drainage canals **fully** penetrate the aquifer, and the water level in the canals is maintained at + 3 m, relative to the impermeable bottom of the aquifer. The distance between the ground surface and the bottom of the drainage canals is 10 m. The daily recharge from precipitation and irrigation is 0.015 m/day. What is the optimal distance between the drainage canals?

e. What is the discharge rate per unit width of the aquifer to the drainage canals?

2. The transmissivity and storativity of a confined aquifer are $T = 1000$ m^2/day and $S = 0.0001$ respectively. An observation well is located at a distance of $r = 500$ m from a pumping well. For a pumping period of $t = 220$ minutes, **calculate** (using the Theis solution, see the graph of the Well-function on the formula sheet)

   a. the drawdown at the observation well if the pumping rate is given by $Q = 1000$ m^3/day

   b. the pumping rate required to provide a drawdown of 1 m in the observation well after $t = 220$ minutes

   c. The Theis solution is based on several assumptions. List three of them, and indicate their relevance.

   d. The overall thickness of the confined aquifer is $B = 100$ m, while the effective porosity is $n = 0.4$. The density and viscosity of the groundwater are $\rho = 1000$ kg/m^3 and $\mu = 0,001$ kg/(ms) respectively. The Carman-Kozeny-Bear relation is given by

$$K = \left(\frac{\rho g}{\mu}\right) \frac{n^3}{(1-n)^2} \left(\frac{d_{10}^2}{180}\right)$$

   Determine the grain diameter d_{10} of the sediment and the corresponding intrinsic permeability.

3. Judge whether the following statements are true or not: motivate your answer!!

   a. In a heterogeneous conductivity field, the hydraulic head contours are closer to each other in areas where the hydraulic conductivity is lower

   b. A no-flow boundary is not a streamline

c. Consider the storativity S of a confined aquifer. The contribution of the compressibility of the fluid can almost always be disregarded compared to the contribution of the compressibility of the sediment material

d. "Quicksand" or liquefaction occurs when the vertical effective stress exceeds the pore pressure

e. Theis solution for pumping test analysis can be represented by

$$h_0 - h(r,t) = \frac{Q}{4\pi T}(-\gamma - \ln u)$$

if $u >> 1$

f. If the thickness of a confined aquifer varies in the horizontal x-direction, the corresponding specific discharge is constant in that direction.

4. Consider the 'idealized' steady-state seawater intrusion problem depicted in Figure 1. The transition zone between fresh and salt water can be considered as a 'sharp interface'. The relative density difference between fresh and salt water is

$$\varepsilon = \frac{\rho_s - \rho_f}{\rho_f},$$

where ρ_s and ρ_f denote the salt water density and the fresh water density, respectively.

a. According to Ghyben-Herzberg, the following relationship holds (see Figure 1 for details)

$$h(x) = \frac{1}{\varepsilon}\,\phi(x)$$

Give a derivation of this relationship based on physical arguments.

b. According to prof. dr. C. de Vries (Vrije Universiteit A'dam), ϕ_{max} is approximately 5 m in the coastal areas of the Netherlands. What is the corresponding h_{max}, if $\rho_f = 1000$ kg/m^3 and $\rho_s = 1025$ kg/m^3?

c. Assuming that the flow of fresh water towards the sea in the phreatic aquifer is essentially horizontal, continuity requires that

$$Q_0 + N\,x = -K(h(x) + \phi(x))\,\frac{\partial \phi(x)}{\partial x},$$

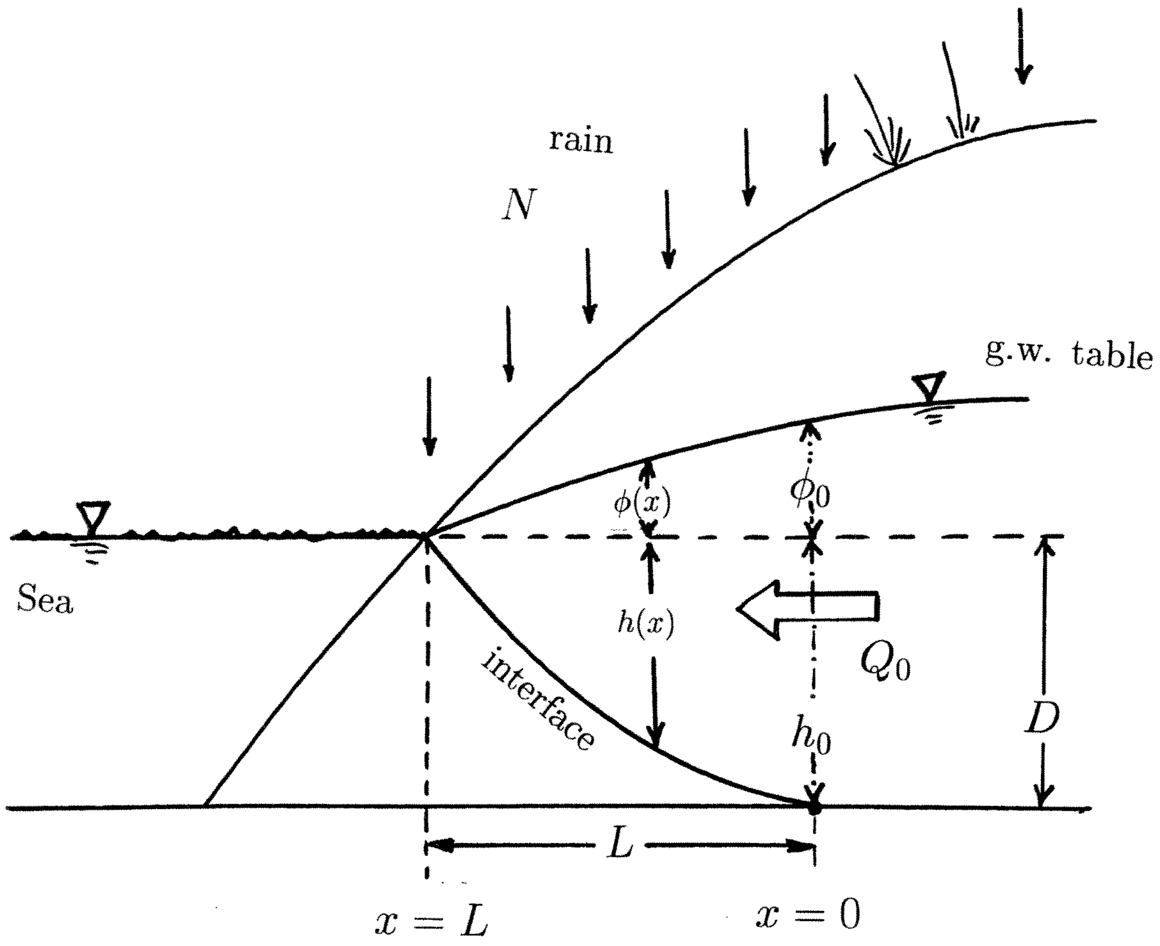

Figure 1. Idealized seawater intrusion situation

where Q_0 is the fresh water discharge at $x = 0$, see Figure 1, and K is the hydraulic conductivity of the sediment. Using the Ghyben-Herzberg relationship, this can we written as

$$Q_0 + N\,x = -K\left(1 + \frac{1}{\varepsilon}\right)\,\phi\,\frac{\partial \phi}{\partial x}, \tag{1}$$

where of course $\phi = \phi(x)$. Show, by derivation, that the hydraulic head distribution satisfies

$$\phi_0^2 - \phi^2(x) = \frac{2Q_0\,x + N\,x^2}{K(1+1/\varepsilon)}, \quad \text{for } 0 < x < L$$

Hint: Integrate equation (1) and use the boundary condition at $x = 0$.

d. In Figure 1, L denotes the length of the seawater intrusion. At $x = L$ we have $\phi(L) = 0$. Moreover, at $x = 0$ we have $\phi(0) = \phi_0 = \varepsilon\,D$ (because of the Ghyben-Herzberg relationship), see again Figure 1. Show that the freshwater discharge Q_0 at $x = 0$ is given by

$$Q_0 = (\varepsilon^2 + \varepsilon)\,\frac{KD^2}{2L} - \frac{NL}{2}$$

e. This last expression, i.e. the relation between Q_0 and L is very important in controlling seawater intrusion in coastal aquifers. It indicates that this length L is a 'decision variable'. Let $K = 10$ m/day, $N = 0.001$ m/day, $D = 50$ m, and $Q_0 = 1.5$ m^2/day. The relative density difference is $\varepsilon = 0.025$. Compute the corresponding seawater intrusion length L.

f. By artificial recharge of fresh water in the dune area, the intrusion length can be decreased. If we want to decrease the intrusion length L by a factor 2, i.e. compared to the length found under e., how much must we increase Q_0?

g. Consider the special case $Q_0 = 0$. Compute the corresponding intrusion length, and explain what $Q_0 = 0$ hydrogeologically implies.

....The End....

Formula sheet

Transient flow

$$h = h(x, y) - \frac{Q}{4\pi T} W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-m}}{m}\, dm$$

Approximation of $W(u)$ (truncated series)

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{4} + \frac{u^3}{18} - \frac{u^4}{96} \ldots$$

$\gamma = 0.5772157...$ Euler's constant

$$u = \frac{r^2 S}{4T(t - t_0)}$$

Radial flow in a confined aquifer

$$h(r) = \frac{Q}{2\pi KB} \ln r + C_1$$

Radial flow in an unconfined aquifer

$$h^2(r) = \frac{Q}{\pi K} \ln r + C_2$$

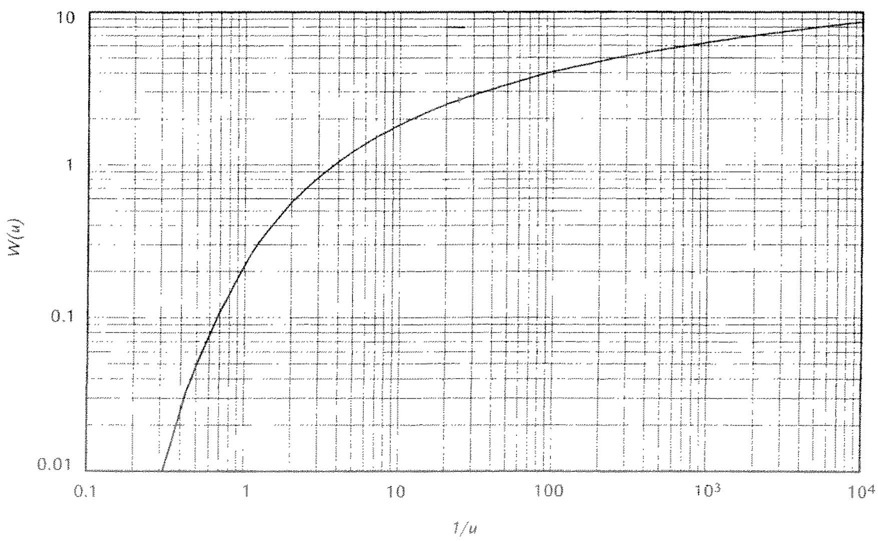

Figure 7.2 Well function $W(u)$ vs. $1/u$ for the Theis solution. Both are dimensionless numbers.