Principles of Groundwater Flow

November, 2012, 9:00-12:00

1. Because salt water intrusion has the potential to impair the quality of coastal aquifers, studies have continued for more than a century to understand the behavior of freshwater/saltwater systems and their interfaces. Nowadays, powerful computer codes (models) are available to simulate density dependent flow and transport. Nevertheless, the 'early' historical findings in the field of density-dependent flow of groundwater are still of great value to gain insight in freshwater/saltwater systems. Independently, Ghyben (1889) and Herzberg (1901) formulated a physically based formula to predict the position of the fresh-saltwater interface.

   (a) If in Figure 1, h_f is the distance between sealevel and the position of the water table, and b is the distance between sealevel and the position of the saltwater-toe, derive the expression for the Ghyben-herzberg relation

   $$h_f = \varepsilon b$$

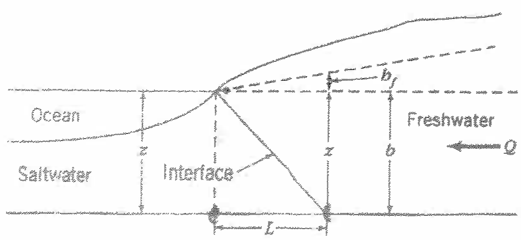
   (b) For seawater intrusion management purposes it is important to know how far the 'toe' of the saltwater-freshwater interface has travelled land-inward. A measure for this intrusion length is

   $$L = \frac{\varepsilon}{2}\frac{kb^2}{Q'},$$

   where k is hydraulic conductivity [m/s], b is the thickness of the freshwater layer relative to sealevel [m], Q' the volumetric flow rate per unit length of the coastline [m^2/s], and ε is the relative density difference,

   $$\varepsilon = \frac{\rho_s - \rho_f}{\rho_f},$$

   where ρ_s denotes the density of the salt(sea)water, and ρ_f the density of the fresh water, see Figure 1. Note that in Figure 1, h_f denotes the distance between sealevel and the position of the groundwater table.



Use the theory of Ghyben-Herzberg to show that this equivalent to

$$L = \frac{1}{2}\frac{h_f kb}{Q'}.$$

(c) Somebody states that the volumetric flow rate Q' at the right-hand side of the saltwater-toe is given by

$$Q' = -k(h_f + b)\frac{d(h_f + b)}{dx}$$

Is this correct or wrong? Motivate your answer.

(d) Let's assume that at the position of the saltwater-toe h_f is very small as compared to b, i.e. $h_f << b$. Show that at the position of the saltwater-toe the hydraulic gradient can be approximated by

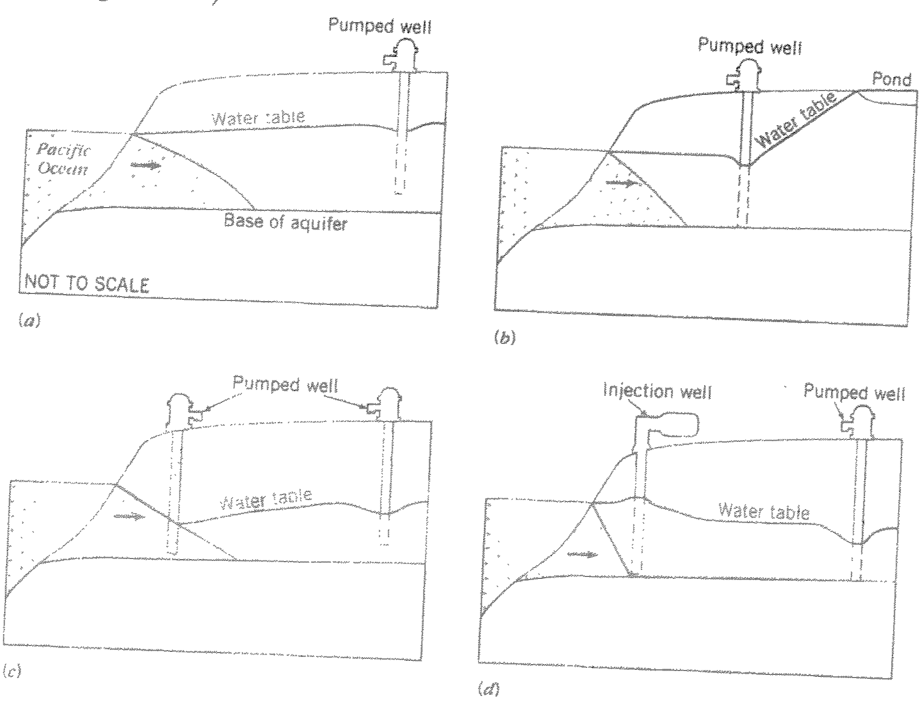
$$\frac{dh_f}{dx} \approx \frac{h_f}{L}$$

Explain why this makes sense in hydrogeological terms.

(e) The water levels in two monitoring wells far from the shore line are 0.5 and 1.0 m above sealevel. The distance between these two wells is 500 m. The hydraulic conductivity is $k = 10$ m/day. The aquifer thickness is 25 m. Compute the volumetric flow rate per unit length of the coastline Q'.

(f) Determine the corresponding saltwater intrusion length L if $\rho_s = 1025$ kg/m^3 and $\rho_f = 1000$ kg/m^3.

(g) Amongst many others, four typical methods can be applied to prevent saltwater intrusion into coastal aquifers. Below are four sketches given of these 'management' methods. Explain why they might be effective and what possible advantages and disadvantages are. (Please use physical, environmental and hydrogeological arguments!)



Methods to prevent saltwater intrusion (from Planert and Williams, 1995)

2. A confined aquifer has a transmissivity $T = 60$ m^3/day and a storativity of $S = 0.0002$. Initially, the hydraulic head distribution is horizontal. A fully penetrating well starts pumping in this aquifer with a rate of $Q = 500$ m^3/day. An observation well is located 30 m away from the pumping well.

   (a) Use Theis solution to predict the drawdown in the observation well, i.e $h_0 - h$, at the following elapsed times: 10 minutes, 1 hour and 5 hours. (Please convert UNITS if necessary!)

   (b) In the same aquifer, a new well is installed with diameter $d = 1.0$ m. The results of the previous pumping test are assumed to be still valid for the aquifer, i.e. S and T. The new well is tested by pumping 150 m^3/day for 10 seconds and then shut off. Predict the drawdown in this well at the following times since the start of pumping: 0, 1, an 10 seconds. (The test starts with a perfectly horizontal hydraulic head distribution.)

   (c) Assume that a pumping test is performed in the vicinity of a river. The river partially penetrates the confined aquifer in which the pumping test is performed. Will this lead to an overestimation or an underestimation of the drawdown compared to the (ideal) Theis-drawdown in the absence of this river. Motivate your answer!

   (d) The Well-function in Theis-solution can be approximated by the series expansion

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{4} + \frac{u^3}{18} - \frac{u^4}{96} \cdots$$

   Under the assumption that $u << 1$, this expansion can be approximated by:

$$W(u) \approx (-\gamma - \ln u)$$

   If $u = 1$, compute the relative error between the 'exact' Well-function value (use the graph), and the truncated series approximation.

   (e) Repeat this computation for $u = 0.1$.

   (f) For which value of u is the relative error between the exact Well-function value and the truncated series approximation less than $\approx$5%?

3. Answer the following questions and motivate your answer!

   (a) The wet bulk density ρ_b of a porous medium is given by

   $$\rho_b = n\rho_f + (1-n)\rho_s$$

   where n denotes porosity, ρ_f fluid density and ρ_s the density of the solid. Correct or not? Motivate!

   (b) The Analytical Element Method (AEM) is based on a mathematical principle. What is this principle? Explain what the advantages are of the AEM and indicate possible limitations. Hint: compare to the finite difference method in e.g. ModFlow.

   (c) The Italian Professor Gambolati suggested to 'lift' Venice, i.e. to revert subsidence, by injecting seawater in a series of deep wells positioned around Venice. Explain on which principle/law this idea is based in hydrogeological terms.

   (d) It is much easier to store fresh water in a phreatic (unconfined) aquifer, as compared to storage in a confined aquifer. Why?

   (e) During drilling of a borehole, e.g. to install a pumping well, the borehole is filled with water or drilling mud as drilling proceeds. Why is this?

   (f) Streamlines are always perpendicular to constant-head lines. True or not? Motivate!

   (g) Consider the island 'Vlieland' in the north of the Netherlands. Underneath this island, a fresh-saltwater interface exists. In September the maximum depth of the interface is less as compared the month of April. True or not? Motivate!

4. Consider the vertical cross section of a permeable dam ('dike') on top of a horizontal confining clay layer. The width of the dam is $L = 20$ m. At the left-hand side of the dam, a lake is present. The waterlevel, relative to the impermeable base, in the lake is constant: $h_0 = 5$ m. At the right-hand side, a drainage canal is present. The water level in the canal is also constant: $h_L = 2$ m relative to the impermeable base. The hydraulic conductivity of the dam is $k = 0.01$ m/day. The dam is located in the Netherlands, so there is a constant and uniform

recharge (rain): $N = 0.002$ m/day. The governing differential equation for steady phreatic flow in the dam is given by

$$k\frac{d}{dx}\left(h\,\frac{dh}{dx}\right) + N = 0$$

(a) Derive the an expression for the position of the water table as a function of x in the dam, i.e. $h = h(x)$.

(b) Compute the position of the water divide in the dam.

(c) What is the minimum height of the dam such that it acts as a hydraulic barrier between the lake and the canal?

(d) Determine the minimum value of N such that no water divide exists in the dam.

.... **The end** ....

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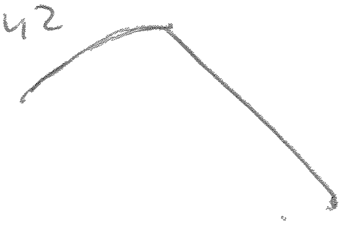

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