2 xamen online

Exam Theoretical Seismology (GEO4-1408)

November 10, 2011; 13:30-16:30

1. The representation theorem is given by:

$$\begin{aligned} u_n(\bar{x},t) &= \int_{-\infty}^{\infty} d\tau \int_V G_{ni}(\bar{x},t-\tau;\bar{\xi},0) f_i(\bar{\xi},\tau) dV(\bar{\xi}) \\ &+ \int_{-\infty}^{\infty} d\tau \int_S [G_{ni}(\bar{x},t-\tau;\bar{\xi},0) T_i^{\bar{u}}(\bar{\xi},\tau) \\ &\qquad - u_i(\bar{\xi},\tau) n_j c_{ijpq} G_{np,q}(\bar{x},t-\tau;\bar{\xi},0)] dS(\bar{\xi}) \end{aligned}$$

(a) Explain each of the terms in all detail.
(b) In case of slip $\Delta\bar{u}$ along a fault Σ with normal $\hat{\nu}$, this equation can be reduced to:

$$u_n(\bar{x},t) = \int_{-\infty}^{\infty} d\tau \int_{\Sigma} \Delta u_i(\bar{\xi},\tau) \nu_j c_{ijpq} G_{np,q}(\bar{x},t-\tau;\bar{\xi},0) d\Sigma$$

Under which conditions reduces the equation in (a) to the one in (b)?
Show how the equation in (b) can be derived from the one in (a).
(c) The equation in (b) can be rewritten as

$$u_n(\bar{x},t) = \int_{\Sigma} m_{pq} * G_{np,q}\, d\Sigma$$

where m_{pq} is the moment density tensor and $*$ denotes convolution.
For an isotropic medium, give the moment density tensor for slip $\Delta\bar{u}$ along a fault with normal $\hat{\nu}$.
Then, give the moment density tensor for $\Delta\bar{u} = (\Delta u_1, 0, 0)$, and $\hat{\nu} = (0, 1, 0)$.
(d) Green's function for a homogeneous, isotropic, elastic medium is given by:

$$\begin{aligned} G_{np}(\bar{x},t;\bar{\xi},0) &= \frac{1}{4\pi\rho r^3}(3\gamma_n\gamma_p - \delta_{np}) \int_{r/\alpha}^{r/\beta} \tau\delta(t-\tau) d\tau \\ &+ \frac{1}{4\pi\rho\alpha^2}\gamma_n\gamma_p \frac{1}{r}\delta(t-r/\alpha) \\ &+ \frac{1}{4\pi\rho\beta^2}(\delta_{np} - \gamma_n\gamma_p)\frac{1}{r}\delta(t-r/\beta) \end{aligned}$$

Explain the meaning of each of the three terms, and describe their characteristics.
Illustrate the particle motion/radiation pattern for the 2nd and 3rd term.
(e) Determine the time dependence of the first term on the right hand side in (d).
Illustrate this time dependence with a sketch.
(f) The far-field P-wave displacement field in case of a point dislocation source is found to be:

$$u_n^P(\bar{x},t) = \frac{\gamma_n\gamma_p\gamma_q}{4\pi\rho\alpha^3 r} \dot{M}_{pq}(t-r/\alpha)$$

where $\dot{M}_{pq}(t-r/\alpha)$ is the time derivative of the moment tensor.
Where does this time derivative come from?

(g) Use (f) to explain how the P-wave displacement varies as a function of azimuth for the case $\Delta\bar{u} = (\Delta u_1, 0, 0)$ and $\hat{\nu} = (0, 1, 0)$.
Illustrate this by taking various locations for $\bar{x}$: $\bar{x} = (a, 0, 0)$, $\bar{x} = (a, a, 0)$, $\bar{x} = (-a, -a, 0)$ and $\bar{x} = (0, 0, a)$, where a is a constant.

2. Find the travel time T and the horizontal distance X for a ray with horizontal slowness p_x for a model with a seismic velocity that varies as a function of depth: $c(z)$. Starting point and end point of the ray are at the surface.

3. (a) What are inhomogeneous waves? When do they occur?
(b) At the free surface ($z = 0$) of an isotropic medium, an inhomogeneous P-wave is given by

$$\bar{u}^{P,inhom} = \grave{P} \begin{pmatrix} \alpha p \\ 0 \\ i\sqrt{\alpha^2 p^2 - 1} \end{pmatrix} e^{-\omega\sqrt{p^2-\alpha^{-2}}z}\, e^{i\omega(px-t)}$$

and an inhomogeneous S-wave by

$$\bar{u}^{S,inhom} = \grave{S} \begin{pmatrix} i\sqrt{\beta^2 p^2 - 1} \\ 0 \\ -\beta p \end{pmatrix} e^{-\omega\sqrt{p^2-\beta^{-2}}z}\, e^{i\omega(px-t)}$$

What are the boundary conditions that couple these two waves in case of a Rayleigh wave?
So what are the conditions that need to be satisfied? (Full simplification of the equations it not necessary, the first step suffices.)

4. (a) Why are surface waves dispersive in the real Earth?
(b) We know that the relation between group and phase velocity can be written as

$$U = \frac{c}{1 + \frac{T}{c}\frac{dc}{dT}} \; .$$

Assume that

$$c = c_o + \alpha\frac{dc}{dT} \; .$$

If $\alpha > 0$, $U < c$.
If $\alpha < 0$, $U > c$.
For Love waves we saw that

$$\omega^2 I_1 = k^2 I_2 + I_3$$

and

$$U = \frac{I_2}{cI_1} \; .$$

Using this expression, can α be positive? Can α be negative?
For the correct case, sketch the seismogram corresponding to the dispersion of the Love wave.