Exam Theoretical Seismology (GEO4-1408)

January 25, 2017; 9:00-11:30

1. (a) Starting from the equation of motion, show that the displacement equation for infinitesimal motion in an elastic medium is:

$$\rho \ddot{u}_i = f_i + \left(c_{ijkl} u_{k,l}\right)_{,j} .$$

(b) In the absence of body forces, this equation reduces to

$$\rho \ddot{u}_i = \left(c_{ijkl} u_{k,l}\right)_{,j} .$$

For a homogeneous elastic medium, de displacement $\bar{u}$ of a plane wave propagating in the direction $\hat{n}$ with velocity c is given by

$$\bar{u}(\bar{x}, t) = \bar{a} f\left(\frac{\hat{n} \cdot \bar{x}}{c} - t\right)$$

with f a twice differentiable function. Show that substitution of this plane wave into the wave equation yields an eigenvalue problem in the form $m_{ik} a_k = \lambda a_i$. Give expressions for λ and m_{ik}.

(c) What does m_{ik} become in an isotropic medium?

(d) Describe what happens when the medium becomes radially anisotropic. It is not necessary to derive the corresponding equations.

2. The elastodynamic Green's function for a homogeneous, isotropic, elastic medium is given by:

$$\begin{aligned} G_{np}(\bar{x}, t; \bar{\xi}, 0) = & \frac{1}{4\pi\rho r^3}\left(3\gamma_n\gamma_p - \delta_{np}\right)\int_{r/\alpha}^{r/\beta} \tau\delta\left(t-\tau\right) d\tau \\ & + \frac{1}{4\pi\rho\alpha^2}\gamma_n\gamma_p \frac{1}{r}\delta\left(t - r/\alpha\right) \\ & + \frac{1}{4\pi\rho\beta^2}\left(\delta_{np} - \gamma_n\gamma_p\right)\frac{1}{r}\delta\left(t - r/\beta\right). \end{aligned}$$

(a) Explain the meaning and describe the characteristics of each of the three terms.

(b) Illustrate the particle motion for the second and third terms, explain your illustration.

(c) Determine the time dependence of the first term, and illustrate the time dependence with a sketch.

3. Derive the eikonal and transport equations for the scalar wave equation $\nabla^2\phi - c^{-2}\ddot{\phi} = 0$. Do this by applying the (temporal) Fourier transform to the wave equation, and taking solutions of the form $\phi(\bar{x}, \omega) = \phi_0(\omega) A(\bar{x}) e^{i\omega T(\bar{x})}$.
Under which conditions is the eikonal equation a good approximation?

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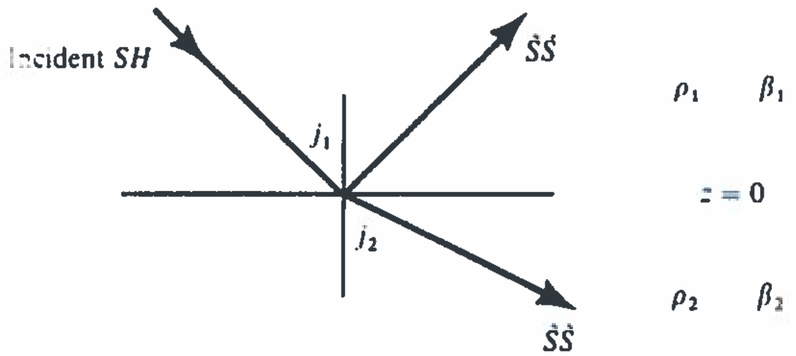
4. Consider an SH-wave with frequency ωincident at an interface $z = 0$. The amplitude of the incident wave is S. The SH-reflection coefficient is $\grave{S}\acute{S}$, the transmission coefficient is $\grave{S}\grave{S}$. The shear velocities and densities are indicated in the figure. The x-direction is positive to the right, z is positive downwards.



(a) Give an expression of the displacement $\overline{u}^{inc}(\overline{x}, t)$ of the incident wave. Also give the displacement of the reflected ($\overline{u}^{refl}$) and transmitted ($\overline{u}^{trans}$) waves.
(b) Show that Snell's law is obtained from the continuity of displacement at the interface.
(c) When does the transmitted S-wave become inhomogeneous? Describe the differences between the inhomogenous wave and a normal travelling transmitted wave.

5. (a) Why are surface waves dispersive in the real Earth?
(b) Sketch the phase velocity as a function of period for a fundamental mode Love wave for a layer with shear velocity β_1 over a half space with shear velocity β_2 ($> \beta_1$).
(c) Draw a second curve in the same plot for a layer with a larger thickness. Explain the difference.
(d) Derive the equation relating the group velocity U to the phase velocity c as a function of wavenumber.