tussentoets (40% 2010-2011)

GEO4-4417 Unsaturated zone hydrology (2010-2011)

You may answer the questions in English or Dutch.
Present arguments with all your answers!
Total number of credits to be earned: 80 (= 10 judicial points).

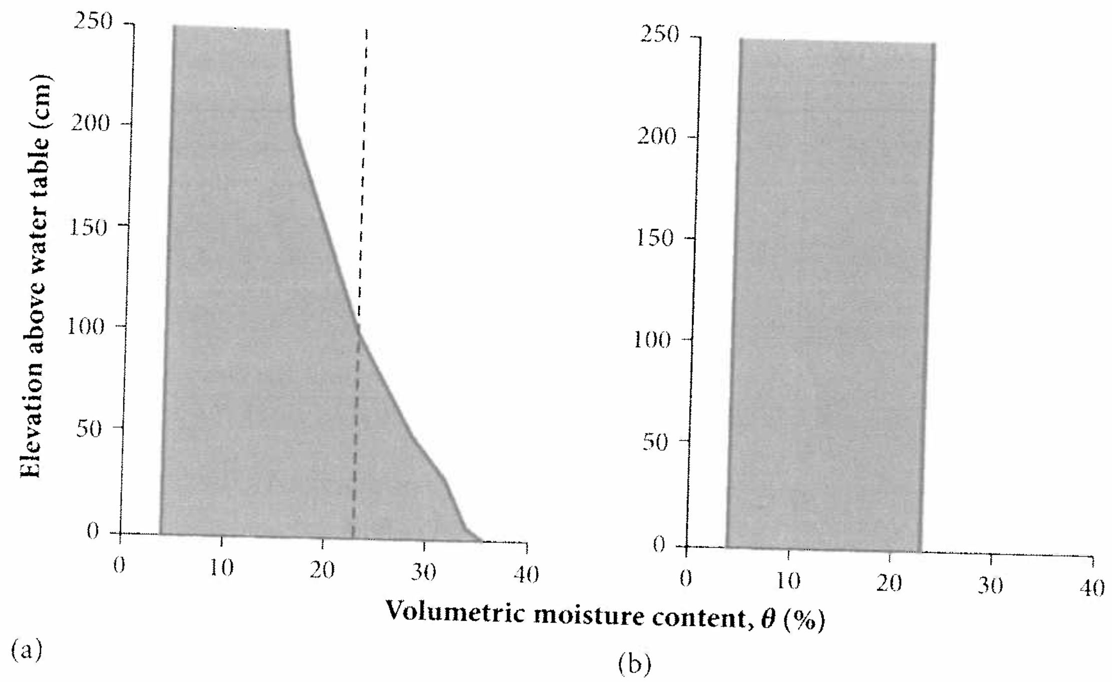
Question 1 (15 credits)

The FAO Penman-Monteith equation is given as:

$$E_{rc} = \frac{0.408\,\Delta\,(R_n - G) + \gamma \dfrac{900}{T+273} u_2\,(e_s - e_a)}{\Delta + \gamma(1 + 0.34\,u_2)}$$

a. Give the correct names for (not more than) 5 parameters in the above equation.
b. Give the units of measurement for these 5 parameters.
c. Which parameters in the above equation depend on temperature?

Question 2 (5 credits)



A physical model (a) and an approximation model (b) of the available soil water for plants with a water table at 2.5 m below the land surface – the approximation model uses $\theta_{pF=2.0} - \theta_{pF=4.2} = 23\% - 4\% = 19\% = 7.6$ cm water for a root depth of 40 cm

By stating the necessary adjustments to the above figures, clearly explain what *pF*-value should be used for field capacity when approximating the available soil water for plants for a situation with a water table at 1 m below the land surface.

Question 3 (10 credits)

Australian soil physicist John R. Philip (1927-1999) described the gradual decline of the ponded infiltration rate f as:

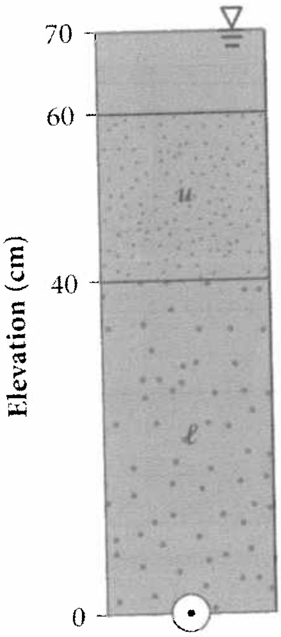
$$f = \tfrac{1}{2} S t^{-\frac{1}{2}} + K$$

The cumulative infiltration F for ponded infiltration can then be described as:

$$F = S t^{\frac{1}{2}} + Kt$$

Clearly describe two techniques to determine both S and K from one or both of the above equations.

Question 4 (10 credits)

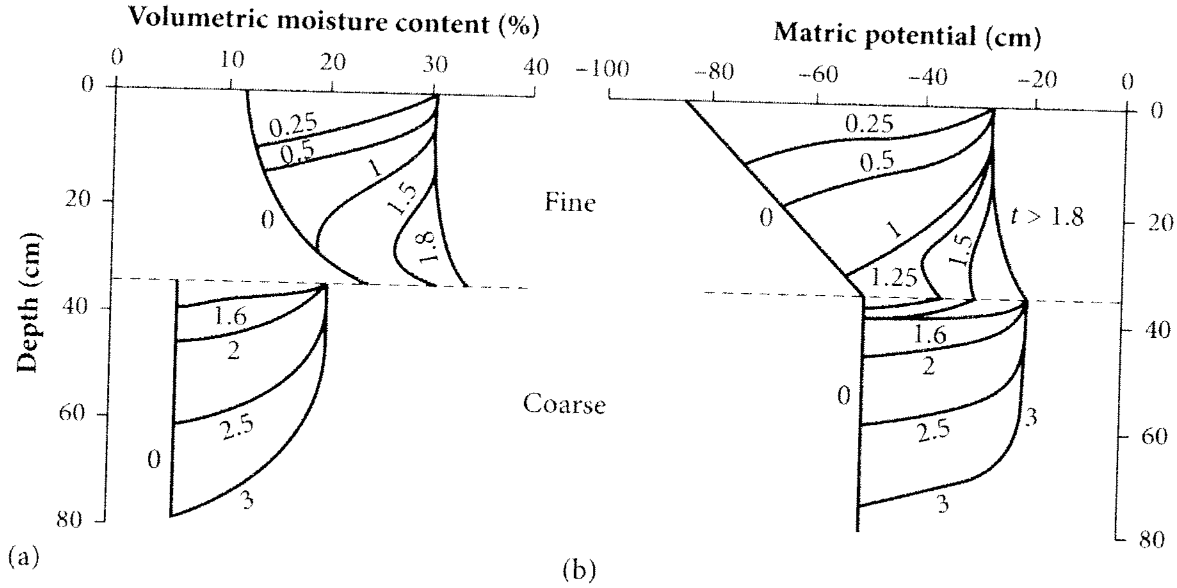


The soil profile of an agricultural field consists of an upper layer of 20 cm and a lower layer of 40 cm. A drainage pipe is located at the bottom of the lower layer. The water level in the field is maintained at 10 cm above the soil surface. The saturated hydraulic conductivity of the upper layer is $\frac{3}{8}$ times that of the lower layer.

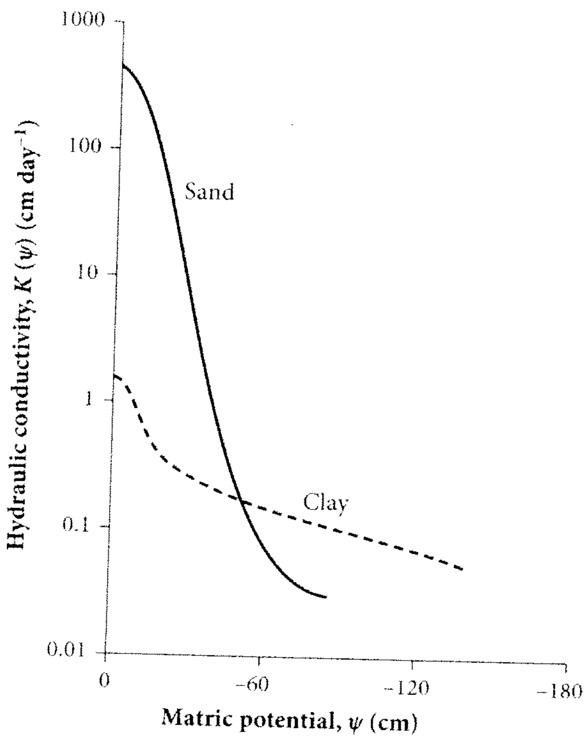
a. Draw a potential diagram with the elevation head z, pressure head h and hydraulic head H on mm paper.

b. Name two assumptions that need to be made in order to draw the potential diagram.

Question 5 (15 credits)



Profiles of the change with time (hours) of volumetric moisture content (a) and matric potential (b) during a constant rate of water application to a layered soil of fine sand overlying coarse sand (after Vachaud et *al.* 1973)



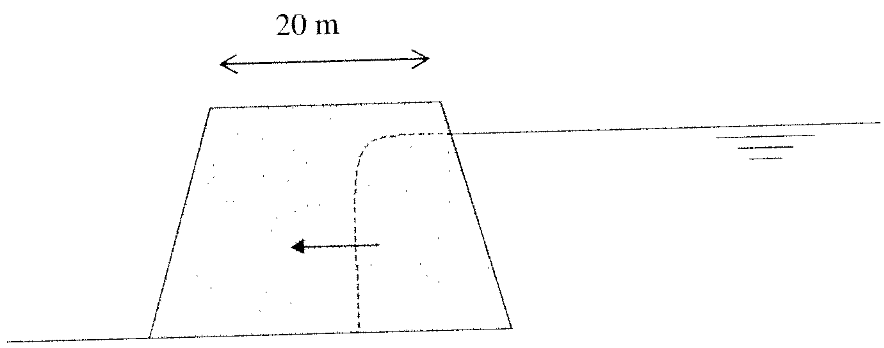
Unsaturated hydraulic conductivity $K(\psi)$ as a function of the matric potential ψ for a sand and clay soil (simplified from Bouma 1977)

a. From the figures on the preceding page, estimate the flux density in m day^{-1} at $t = 1.5$ hours. Clearly state the successive steps you took to answer this question.
b. Describe what happens at the boundary between the fine and coarse sand between 1 and 2 hours.

Percolation through a fine-textured layer that overlies a coarse-textured layer may give rise to the development of a certain type of flow in the coarse-textured layer.

c. Give both a general and a more specific name for this type of water flow in the coarse-textured layer.

Question 6 (15 credits: 6a: 3; 6b: 12)



Consider a river levee of 20 m wide holding water during a high river stage (see the figure above). If the water level of the river remains high for a long period of time, the level will take up a lot of water due to (horizontal) filtration. If the levee becomes fully saturated (the filtration front reaches the other side of the levee), the levee may become unstable and break. The time it takes for the levee to get saturated is therefore an important safety parameter. Intuitively one would assess that the time to full saturation depends on the soil physical properties of the levee and the levee's initial soil moisture content at the time the high water level first occurs. We can try to calculate the time to full saturation with horizontal filtration theory. To do this we need the sorptivity of the soil inside the levee for a given initial moisture content θ_i. A well known relationship is the one given by Brutsaert (2005):

$$S = (\theta_s - \theta_i)\sqrt{\frac{2D_{sat}}{m+1}}, \text{ with}$$

S = the sorptivity of the soil inside the levee in m day$^{-1/2}$,
θ_s = the saturated moisture content = 0.43,
m = the Van Genuchten parameter = 2, and
D_{sat} = the diffusivity near saturation = 200 m^2 day^{-1}.

Answer the following questions:

a. Calculate the sorptivity for the following initial soil moisture contents ($\theta_i = 0.1, 0.2, 0.3, 0.4$)!
b. Assume pure horizontal filtration and consider that the length of infiltration front is given by $L = I/(\theta_s - \theta_i)$, with I the cumulative amount of filtration. Calculate the time it takes from the beginning of a flood for the levee to become fully saturated (the time it takes for the moisture front to travel 20 m) for $\theta_i = 0.1, 0.2, 0.3, 0.4$. What is your (unexpected) conclusion?

Question 7 (10 credits: 7a: 3; 7b: 1; 7c: 1; 7d: 2; 7e: 3)

a. Consider a two-layered soil, sand on top of clay. When describing unsaturated flow in this soil, which of the two forms of the Richards' equation do you prefer: the diffusivity form or the capacitance form? Explain why.
b. Name one advantage of using a finite element method for solving Richards' equation when compared to using a finite difference method.
c. Name one disadvantage of using a finite element method for solving Richards' equation when compared to using a finite difference method.
d. Name two advantages of using an implicit scheme over an explicit scheme when solving Richards' equation with finite differences.
e. Name three types of boundary conditions for 1D soil water flow. Provide an example for each type.

References

Bouma, J. (1977). Soil survey and the study of water in the unsaturated zone. Soil Survey Paper 13. Netherlands Soil Survey Institute, Wageningen, 106 pp.

Brutsaert, W. (2005). Hydrology: An Introduction. Cambridge University Press, 605 pp. doi: 10.2277/0521824796.

Vachaud, G., Vauclin, M., Khanji, D. and Wakil, M. (1973). Effects of air pressure on water flow in an unsaturated stratified vertical column of soil. *Water Resources Research*, 9, 160-173.

ANSWERS

1. See pages 41, 44, and 45 in Hendriks (2010). Temperature dependent parameters: name three of the following variables: E_{rc}, Δ, R_n, T, and e_s.

2. See pages 157 to 159 in Hendriks (2010): Reduce the figures (a) and (b) to show only the shaded area between the water table and 100 cm above the water table. Then, to approximate the shaded area of this new figure (a) by an approximation model (b), the right-hand side of the given figure (b) - which equals the broken, vertical line in the given figure (a) - must be shifted to the right, to a θ-value of 29%, thus to a lower pF-value of (log 51 cm =) 1.7 instead of 2.0.

3. See pages 180 to 182 in Hendriks (2010), and in the course syllabus: the 'Infiltration exercise using Excel', and Exercise 2.3.

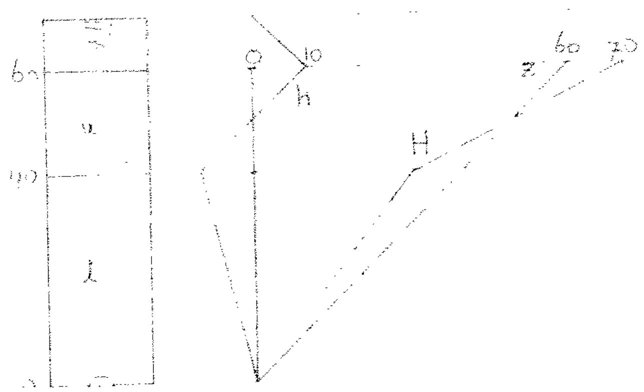
4a. Continuity: $q_u = q_l \qquad -K_u i_l = -K_l i_l \qquad \frac{K_u}{K_l} = \frac{3}{8} = \frac{i_l}{i_u}$

$$H_{60} = z_{60} + h_{60} = 60 + 10 = 70 \qquad H_0 = z_0 + h_0 = 0 + 0 = 0$$

$$i_l = \frac{H_0 - H_{40}}{z_0 - z_{40}} = \frac{0 - H_{40}}{0 - 40} = \frac{H_{40}}{40} \qquad i_u = \frac{H_{40} - H_{60}}{z_{40} - z_{60}} = \frac{H_{40} - 70}{40 - 60} = \frac{H_{40} - 70}{-20}$$

$$\frac{i_l}{i_u} = \frac{3}{8} = \frac{\left(\frac{H_{40}}{40}\right)}{\left(\frac{H_{40} - 70}{-20}\right)} \qquad H_{40} = 30\,\text{cm} \qquad i_l = \frac{30}{40} = \frac{3}{4} \qquad i_u = \frac{30 - 70}{-20} = 2$$

The line for z is a simple diagonal, and the lines for H and h can be drawn:



4b. Name two of the following three:

- The negative water pressure must be less negative than the air entry value (stated as a negative number) for both layers, otherwise the hydraulic conductivity will not be equal to the (constant) saturated hydraulic conductivity, but a function of the negative pressure head (matric head).
- The pressure head in the drainage pipe is zero.
- For the flow process to be maintained there must be continuity of water flow, no stagnation (as shown in the upper figure on page 3).

5a. At t = 1.5 hours, the pressure head is approximately constant at −28 cm and almost independent of depth. The latter means that the hydraulic gradient equals the gradient of the elevation head and thus that the hydraulic gradient equals 1. The hydraulic conductivity at a pressure head (matric potential) of −28 cm for a sandy soil equals some 80 cm day^{-1} as is evident from the lower figure on page 3. Since the hydraulic gradient = 1, the flux density = hydraulic conductivity = approximately 80 cm day^{-1}. (Taking the pressure head as approximately constant at −30 cm, the hydraulic conductivity will be in the order of 10 cm day^{-1}.)

5b. There is a build-up of water above the boundary of the two layers and thus stagnation of the percolating water. Only after the water content or negative pressure head reaches a threshold value can water be transferred to the lower layer. (This threshold value is determined by the overall predominant pore size and associated negative pressure head at the top of the lower layer.)

6a. $S = (\theta_s - \theta_i)\sqrt{\frac{2D_{\text{sat}}}{m+1}} = (0.43 - \theta_i)\sqrt{\frac{2\times 200}{3}} = (0.43 - \theta_i)\cdot 11.547$

This yields:

Theta = 0.1	S = 3.811
0.2	2.656
0.3	1.501
0.4	0.346

6b. Cumulative horizontal filtration is given by: $I = S\sqrt{t}\ \ (m)$. The time it takes to travel 20 m for θ_i = 0.1 is given by:

$$20 = L = I/(\theta_s - \theta_i) = S\sqrt{t}\,/(\theta_s - \theta_i)$$

$$\Rightarrow t = \left(\frac{20(\theta_s - \theta_i)}{S}\right)^2 = \left(\frac{20\times 0.33}{3.810}\right)^2 = 3 \text{ days}$$

For other initial moisture contents it is also calculated at 3 days! This leads to the unexpected conclusion that the time for the levee to saturate is only a function of the soil properties and not of the initial saturation! This can be seen by further developing the equation of time to saturation:

$$t=\left(\frac{L(\theta_s-\theta_i)}{S}\right)^2=\left(\frac{L}{\sqrt{\frac{2D_{sat}}{m+1}}}\right)^2=\frac{(m+1)L^2}{2D_{sat}}=\frac{3\cdot 20^2}{2\cdot 200}=3 \text{ days} = \text{constant}$$

7a. Whereas matric head h is continuous across boundaries between two soil physical units, soil moisture is not. So in terms of obtaining solutions, it is easier to work with the capacitance form in that case.

7b. The finite element method allows more spatial flexibility to be build into the model (for instance, near a sink or source of water; when attempting to build in spatial flexibility using a finite difference method awkward elongated grids evolve).

7c. The mathematics in finite element models is far more complicated than the mathematics in finite difference models.

7d. Name two of the following:

- An implicit scheme (which also uses neighbouring node values during the same time step) is much more stable.
- The error is smaller than for explicit schemes.
- Implicit schemes generally handle saturated situations better than explicit schemes.

7e. The following types:

- Dirichlet: constant head/moisture at the top or bottom of a soil column
- Neumann: evaporation or precipitation flux at the top of a soil column.
- Cauchy: drainage to a ditch or free gravity drainage at the bottom of a column.

References

Course syllabus Unsaturated zone hydrology GEO4-4417, version 2010-2011.

Hendriks, M.R. (2010). Introduction to Physical Hydrology. Oxford University Press, 331 pp. http://ukcatalogue.oup.com/product/9780199296842.do