Re-take Exam Tectonophysics, March 7 2017, 17:00-20:00

1. *Switch off your smartphone (-watch) and put it out of sight*
2. *Not allowed: Head- or earphones, notes, books*
3. *Allowed: graphical calculator, pencils, pens, ruler, compass*
4. *Answer every question (and just the question) as precisely and concise as possible*
5. *You are allowed to leave the room one hour after the test has started (late comers will be allowed in during the first hour).*

Assignment 1. Flexure of a broken elastic lithosphere.

The elastic flexure equation for a uniform plate with zero in-plane force is given by

$$D\frac{\mathrm{d}^4 w}{\mathrm{d}x^4}+(\rho_a-\rho_i)gw=q(x) \tag{1}$$

(a) What is the meaning and physical dimension of D, w, x, ρ_a, ρ_i, g, and q?

(b) What is the physical dimension of $\mathrm{d}^4 w/\mathrm{d}x^4$?

Volcanic activity may have weakened the Hawaii-Emperor chain to the extent that the Pacific plate can be considered to be broken beneath the chain ($x=0$). The Green's function of (1) for a broken plate is

$$G(x)=\frac{k\mathrm{e}^{-kx}\cos kx}{g(\rho_a-\rho_i)} \qquad x\geq 0 \qquad k=\left[\frac{(\rho_a-\rho_i)g}{4D}\right]^{\frac{1}{4}} \tag{2}$$

(c) Use a small sketch to explain why the deflection of the plate due to a surface load $q(x)=\delta(x-y)$ is equal to

$$G(x,y)=\frac{k\mathrm{e}^{-k(x-y)}\cos k(x-y)}{g(\rho_a-\rho_i)} \qquad x\geq y \tag{3}$$

(d) Show that k has the physical dimension of a wavenumber.

(e) Use a small sketch to explain why the surface deflection $w(x)$ due to a surface load

$$q(x)=\begin{cases} 0 & |x|>\dfrac{W}{2} \\ \rho_c g h_0 & x\leq\dfrac{W}{2} \end{cases}$$

should be equal to

$$w(x)=\frac{\rho_c g h_0 k}{g(\rho_a-\rho_i)}\int_{-W/2}^{W/2}\mathrm{e}^{-k(x-y)}\cos k(x-y)\,\mathrm{d}y \qquad x\geq\frac{W}{2} \tag{4}$$

(f) Integrate (4) to find the solution $w(x)$ for $x\geq W/2$.

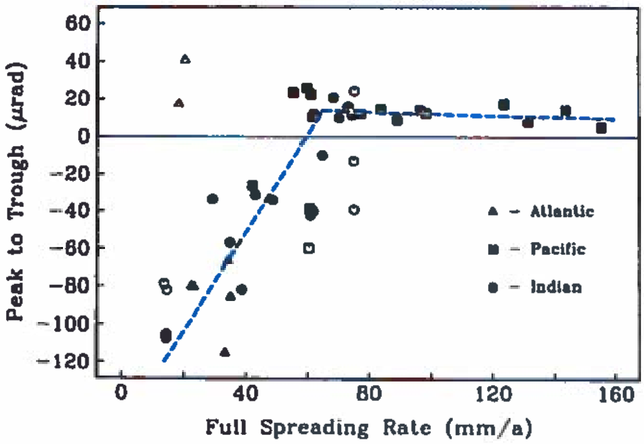

Figure 1. Peak to trough vertical deflection of the geoid over 44 ridge axes versus full spreading rate from Small and Sandwell (1989). Symbol shape indicates major ocean. Open symbols are "anomalous" ridges that are either shallow than 2 km or deeper than 2.9 km. Peak to trough amplitudes are usually negative when highly variable for rates less than 65 mm/yr while at higher rates the amplitudes are positive and uniform. Dashed line shows an abrupt change in slope at 65 mm/yr.

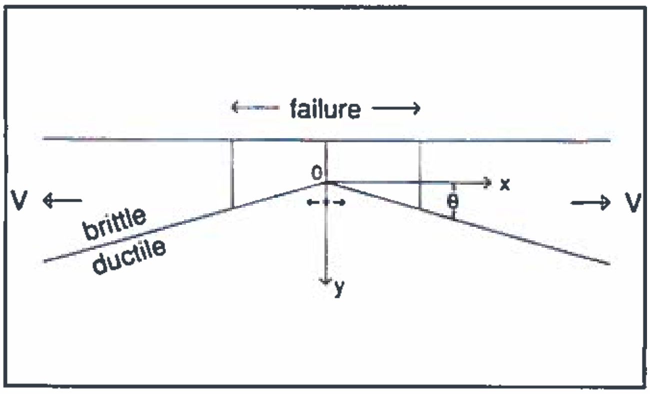

Figure 2. Two triangular shaped rigid plates move away from each other with constant velocity V over a fluid half-space with uniform viscosity.

Assignment 2. Deformation at ridges

Figure 1 is reproduced from Chen and Morgan ("*Rift Valley/No Rift Valley Transition at Mid-Ocean Ridges*", Journal of Geophysical Research, 1990).

(a) What does "Peak to Trough (μrad)" on the vertical axis mean, i.e., use the information in the caption to explain what quantity is plotted here and use a sketch to elucidate its physical meaning.

(b) The authors of the paper claim that the data in Figure 1 show a change in isostatic compensation mechanism. They set up a model to study this change. The model geometry is shown in Figure 2. Chen and Morgan derive the steady state solution for the flow field below the rigid plates:

$$u_x = U\left[\arctan\left(\frac{x}{y}\right) - \frac{xy}{x^2+y^2}\right] \qquad \text{where } U = \frac{V}{\pi/2 - \theta - \sin\theta\cos\theta} \qquad (5)$$
$$u_y = U\left[\sin^2\theta - \frac{y^2}{x^2+y^2}\right]$$

Give and motivate the boundary conditions at $x = 0$ and along the base of the rigid plate for which they derive solution (5) and demonstrate that both boundary conditions come out of (5) along these boundaries. (*Note: expressions at the end of this exam may be useful*)

(c) Explain (in words and schematic mathematical steps, no full derivation required) how solution (5) is used to explain the knick point in Figure 1. Start with the deformation gradient tensor and how it follows from (5).

Assignment 3. Slab dip.

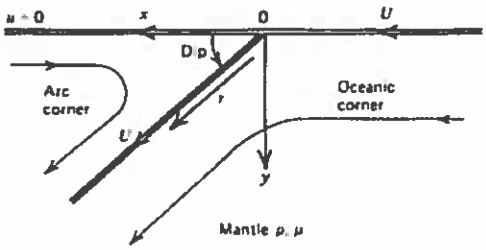

Figure 3

In class we derived the hydrodynamic pressure on an oceanic slab that results from the subduction velocity U of the slab into the asthenosphere (viscosity μ). The total lifting pressure resulting from flow in the arc corner and the oceanic corner is (Figure 3):

$$P = C\frac{\mu U}{r} \tag{6}$$

where $C(\alpha)$ is a constant depending on the dip angle α. Consider a slab with a uniform density contrast $\Delta\rho$ with respect to the asthenosphere, thickness H and length L, and that has a stable dip angle α ($30° \leq \alpha \leq 60°$). Derive an expression for the density contrast as function of μ, U, and α.

Success!!

Auxiliary relations:

$$\cos^2\beta = \frac{1}{1+\tan^2\beta} \quad \sin^2\beta = \frac{\tan^2\beta}{1+\tan^2\beta} \quad \frac{\pi}{2} - \beta = \arctan\left(\frac{1}{\tan\beta}\right)$$