GEO4-1415 Data processing and inverse theory

Tentamen - 9 Nov 2017 - 17h00-19h30 - RUPPERT-BLAUW

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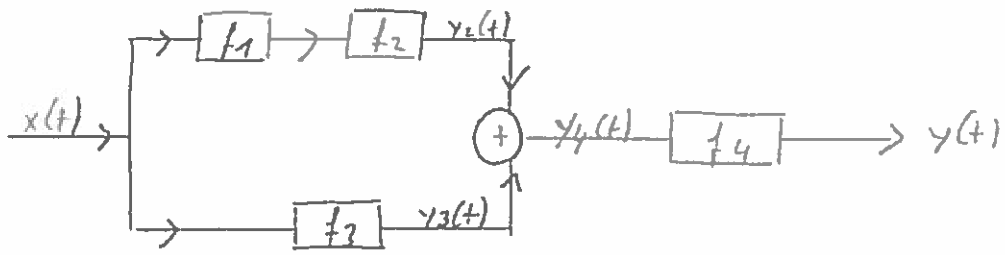
The numbers in () indicate percentage marks for the evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch, English, French, German or Luxembourgish.

1. (10) Consider the following continuous time series

$$x(t) = 3\cos(20\pi t) + 5\sin(50\pi t) + 10\cos(100\pi t) \tag{1}$$

What is the highest frequency in this signal. We sample $x(t)$ with a sampling interval of 0.025 seconds. Do we represent all or part of the sines and cosines correctly? Same question for a sampling interval of 0.005 seconds.

2. (10) Consider the following drawing of a combination of various filters in series and parallel. Give the total impulse response of the filtering operation $f(t)$ as a function of $f_1(t)$, $f_2(t)$, $f_3(t)$ and $f_4(t)$ in the time domain. What is the frequency response $F(\nu)$?



3. (10) We have a time series $x(t)$ sampled with a sampling rate of 1. We average this time series with a running average over 3 points. The output is given by the expression:

$$y_n = \frac{(x_{n-1} + x_n + x_{n+1})}{3} \tag{2}$$

What is the impulse response of this averaging filter. Note that the response is not necessarily causal. What is its frequency response? You don't need to make an explicit calculation of the frequency response.

4. (20) Consider a wavelet $a_t = (\alpha, 0, 0)$. We want to construct a filter, which delays this wavelet by 2 time samples. We call the filter performing this operation f_t. What is the Z-transform of this filter? Deduce the impulse response f_t. We now want to find the same delay filter by constructing a Wiener filter. Set up the problem choosing an appropriate length for the filter f_t and solve it. Do you find the same answer?

5. (20) Two equivalent stochastic solutions may be given for the linear inverse problem $d = Gm$:

$$\tilde{m} = (G^tC_d^{-1}G + C_m^{-1})^{-1}G^tC_d^{-1}d \quad (3)$$
$$= C_mG^t(GC_mG^t + C_d)^{-1}d \quad (4)$$

where C_d and C_m are covarinace operators in the data and model space. respectively.

We know that these 2 solutions are equivalent. In which case would you use which formulation?

For particular values of C_d and C_m, we can give the following solutions:

$$\tilde{m}_1 = (G^tG)^{-1}G^td \quad (5)$$
$$\tilde{m}_2 = G^t(GG^t)^{-1}d \quad (6)$$

Specify C_d and C_m for each solution. Are they equivalent? Justify your answer.

Assume now that C_d is diagonal with a constant value corresponding to the standard deviation of the uniform data uncertainty (i.e. $C_d = \sigma_d^2 I$) Similarly $C_m = \sigma_m^2 I$. Show that this leads to a damped least-squares solution

$$\tilde{m} = (G^tG + \theta^2 I)^{-1}G^td \quad (7)$$
$$= G^t(GG^t + \theta^2 I)^{-1}d \quad (8)$$

Specify θ^2.

6. (30) Solve the following system using singular value decomposition.

$$x - y + z = 3 \quad (9)$$

$$x + y = 2 \quad (10)$$

Calculate the model resolution and the data importance.

Good luck.