# Examination Paper: GEO3-1302

## Continuum mechanics and rheology of the crust and mantle
### PART II (SPIERS)

29-01-2016, 09.00-12.00 hours, Ruppert Blauw

**Note:** an extra half hour is available to students who have registered special needs in advance

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**N.B.**
- This exam paper consists of 5 questions. **Answer 4 of the 5 questions.**
- Take about 45 minutes to answer each question; each question carries equal points
- Answer in English or in Dutch
- Identify all mathematical symbols you use, and give units of quantitative answers
- If you do not understand the English used in a question, raise your hand for help

**Tip:**
- Read the questions carefully and answer what is asked. Check answers before leaving!

**Good Luck!!!**

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**Question 1**

a) Explain briefly the atomic scale basis for linear elastic behaviour in crystalline materials.

b) Define the quantities "Youngs Modulus" and "Poisson's Ratio" used to specify the elastic behaviour of isotropic materials. Illustrate your answer with simple diagrams!!!!!!

c) Write down a set of equations giving the 3-D strain response of an isotropic elastic solid subjected to a stress defined by the principal stresses $\sigma_1$, $\sigma_2$, $\sigma_3$.

d) Use these equations to estimate the strains undergone by a block of dense marble (with Young's modulus $E = 30$ GPa and Poisson's Ratio $\nu = 0.3$) subjected to the stress

$$\sigma_{ij} = \begin{bmatrix} +90 & 0 & 0 \\ 0 & +60 & 0 \\ 0 & 0 & +30 \end{bmatrix} MPa$$

where compression is taken positive.

e) How would you expect $E$, $\nu$ and hence the response of the marble to be modified by the introduction of a population of uniformly distributed, near-spherical pores into the marble? Assume that the pores are filled with an easily compressible gas (e.g. air or $CO_2$) at near atmospheric pressure.

**Question 2**

a) Explain the terms "uniaxial compressive strength" and "failure criterion".

b) Indicate typical values of uniaxial compressive strengths for rocks.

c) Write down the Coulomb criterion for shear failure of dry rock and an expression giving the orientation (angle $\theta$) of the failure plane normal with respect to the principal compressive stress $\sigma_1$ (identify ALL terms appearing and indicate any restrictions on $\theta$).

d) The state of stress near a **sealed** normal fault in an impermeable, 50 m-thick anhydrite caprock formation overlying a reservoir sandstone (depth to top 3 km), into which $CO_2$ is being actively injected for geological storage purposes, is estimated (from numerical modelling) to be evolving with time $t$ (years) in the following manner:

$\sigma_1$(vertical) = 75 MPa (constant)
$\sigma_2$(North-South) = (75 – $a.t$) MPa where $a$ = 0.25 MPa/year
$\sigma_3$(East-West) = (75 – $b.t$) MPa where $b$ = 1.5 MPa/year

These relations show that ($\sigma_1$ -$\sigma_3$) will increase with time $t$ and therefore that there is a risk of fault reactivation. Assuming that these relations for the principal stresses continue to hold, determine when (i.e. at what value of $t$) the fault in the anhydrite will be reactivated. DATA: Sealed faults in anhydrite typically have a cohesive shear strength of 5 MPa and a coefficient of internal friction of 0.5.

e) Sketch a conceptual geological cross-section showing the reservoir, caprock and fault, and state what the consequences of reactivating the fault might be.

**Question 3**

a) Draw a simple block diagram illustrating the essential features of an edge dislocation.
b) Define the terms "dislocation self-energy" and "dislocation density"? Illustrate your answer with appropriate formulas, identifying all terms.
c) Explain what is meant by the term "slip system" and write down an expression for the force (per m) acting on a straight dislocation segment lying in its slip system in a stressed crystal.
d) Explain the operation of the Frank-Read source of dislocations. Make sure you mention the various competing forces involved and the equilibrium condition required to activate such a source. Explain also how this equilibrium condition relates to the plastic yield behaviour of single crystals.
e) The Frank-Read source produces dislocation loops. What does a dislocation loop enclose, and what happens to a small crystal if a dislocation loop expands under stress until it jumps out of the surfaces of the crystal?

**Question 4**

a) Explain what is meant by the term steady state creep, illustrating your answer with sketches of stress-strain and strain-time diagrams.
b) List the main mechanisms by which steady-state creep can occur in crystalline materials, indicating the essential nature of each mechanism.
c) Go on to explain briefly the concept of the deformation mechanism map, illustrating your answer with a schematic labelled diagram.
d) How is such a map constructed?

**Question 5**

a) Explain what is meant by Byerlee's rule and indicate for what portion of the continental lithosphere it is expected to apply.
b) Indicate what deformation mechanisms and constitutive equations are thought to describe the plastic flow behaviour of quartz in the mid-lower continental crust.
c) Explain how you would construct a strength profile for a section of continental **crust** assuming it is quartz-rich and taking a uniform extensional strain rate of say $10^{-15}$ $s^{-1}$?
d) Sketch such a profile for a low and for a high geothermal gradient.
Use your profiles to predict what the effect of geothermal gradient should be on the depth and magnitude of seismicity during extension of the crust.

**Beer Awards Borrel:** All members of the class are invited for the beer awards ceremony and borrel on Thursday 28 April, 17.00. I cannot do it earlier this year due to other commitments. Location follows via e-mail. **CHRIS**