Tentamen Continuummechanica (13-12-2013; docent: Spakman)

Write clearly and provide arguments in derivations and answers. Point scores are given (36 in total).

Useful equations which you may, or may not need: $\frac{d\rho}{dt} + \rho\nabla.\vec{v} = 0$, $\rho\frac{dv_i}{dt} = \frac{\partial\sigma_{ij}}{\partial x_j} + F_i$, $\sigma_{ij}n_j = \sigma_i^{\hat{n}}$

$\frac{d}{dt} = \frac{\partial}{\partial t} + v_k\frac{\partial}{\partial x_k}$, $\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\theta\delta_{ij}$, $\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\epsilon}_{ij} + \upsilon\dot{\theta}\delta_{ij}$, $\rho\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j\partial x_j} + F_i$

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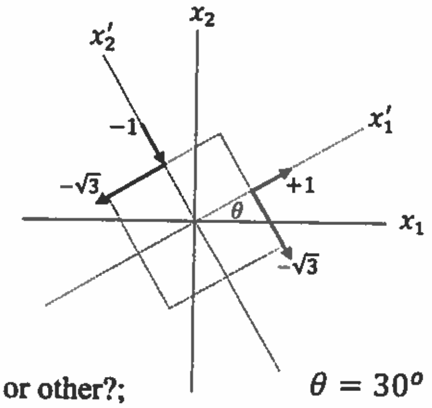
1 a) (2) Describe the physical meaning of the quantities in the Relation of Cauchy.

b) (2) When do we need to use the Relation of Cauchy? Illustrate this with 2 examples.

2) (4) Derive the constitutive equation of an isotropic linear elastic medium from the following two equations: $\sigma_{ij} = C_{ijkl}\varepsilon_{kl}$ and $C_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})$

3) (8) Tractions [MPa] are given on the faces of a tiny square, which is rotated counter-clockwise over 30 degrees as indicated in the figure.



a) (2) Determine the stress tensor in the rotated frame. (explain your answer!)

b) (3) Determine the stress tensor in the $x_1 - x_2$ frame.

c) (2) Determine the traction acting on a surface through the origin with a normal pointing 45 degrees measured counterclockwise with respect to the x_1-axis.

d) (1) What is the nature of the traction under c) [shear, normal, or other?; explain your choice]

4) (7) The following stress field exists in a x_1-x_2-x_3 -frame: $\bar{\sigma} = \begin{bmatrix} -x_3 & x_1x_2 & 0 \\ x_1x_2 & -x_3 & 0 \\ 0 & 0 & -x_3 \end{bmatrix}$

a) (2) Assuming mechanical equilibrium, determine the body force ("volumekracht" in Dutch).

b) (2) Determine the principal stresses ("hoofdspanningen" in Dutch) in P(1,2,3)

c) (3) Determine the principal axes ("hoofdspanningsassen" in Dutch) in P(1,2,3)

5) (4) Derive the Navier-Stokes equation from the General Equation of Motion using an incompressible linearly viscous fluid with constant viscosity η : *(provide explanation on all steps of the derivation)*

$$\rho\left(\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j\partial x_j} + F_i$$

6) (9)

Consider a two-dimensional infinite laminar (parallel) linearly viscous flow of thickness h in x_1 direction. In depth direction the flow is confined between two plates at the depths $x_2=0$ and $x_2=h$, where $x_2=0$ coincides with the upper plate. The viscosity η is constant.

The flow velocity field is given as $\vec{v}(x_1,x_2) = \begin{pmatrix} \frac{\alpha}{2\eta}x_2^2 + x_2 \\ 0 \end{pmatrix}$. The pressure in the fluid is given by $p(x_1,x_2) = \rho g x_2 + \alpha x_1$ and the body force is given by $F_1 = 0, F_2 = \rho g$

a) (2) Determine the strain-rate tensor at (x_1, x_2)

b) (2) Determine the stress tensor at (x_1, x_2)

c) (2) Determine the traction which the flow exerts ("Dutch": uitoefent) on the upper plate

d) (3) Show that the flow field is a solution of the Navier-Stokes equation.