**Tentamen Continuummechanica (14-12-2012; docent: Spakman)**

***Write clearly and provide arguments in derivations and answers. Point scores are given (30 in total).***

Useful equations which you may, or may not need: $\frac{d\rho}{dt} + \rho\nabla.\vec{v} = 0$ , $\rho\frac{dv_i}{dt} = \frac{\partial\sigma_{ij}}{\partial x_j} + F_i$ ,

$\frac{\mathrm{d}}{\mathrm{dt}} = \frac{\partial}{\partial \mathrm{t}} + \mathrm{v_k}\frac{\partial}{\partial \mathrm{x_k}}$ , $\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\theta\delta_{ij}$ , $\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\epsilon}_{ij}$ , $\rho\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j \partial x_j} + F_i$

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**1)** (10 points; ~33% of 30 points)

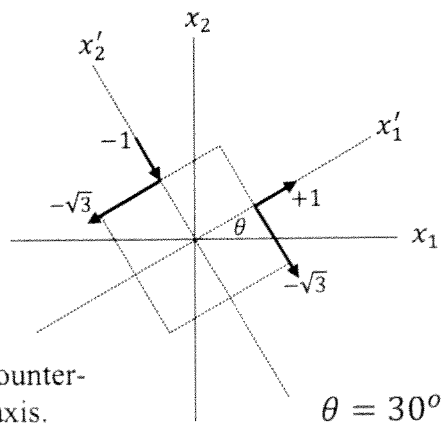
Tractions [MPa] are given on the faces of a tiny square, which is rotated counter-clockwise over 30 degrees as indicated in the figure.

**a)** (2) Determine the stress tensor in the rotated frame. (provide argumentation!)

**b)** (3) Determine the stress tensor in the $x_1 - x_2$ frame

**c)** (2) Determine the traction acting on a surface through the origin with a normal pointing 45 degrees, measured counterclockwise with respect to the $x_1$-axis.

**d)** (3) Determine the stress tensor on a square that is rotated counter-counterclockwise by 45 degrees with respect to the $x_1$-axis.




**2)** (7 points ; 23%)

Assume that the following stress field exists in a $x_1$-$x_2$-$x_3$ -frame: $\bar{\bar{\sigma}} = \begin{bmatrix} x_3 & x_1x_2 & 0 \\ x_1x_2 & x_3 & 0 \\ 0 & 0 & x_3 \end{bmatrix}$

**a)** (2) Determine the body force ("volumekracht" in Dutch).

**b)** (2) Determine the principal stresses ("hoofdspanningen" in Dutch) in P(1,2,3)

**c)** (3) Determine the principal axes ("hoofdspanningsassen" in Dutch) in P(1,2,3)

**3)** (4 points; ~13%)

The postulate of linear viscosity states $\sigma'_{ij} = 2\eta\dot{\varepsilon}'_{ij}$ and relates the deviatoric stress, to viscosity and deviatoric strain rate. Derive from this postulate the constitutive equation

$\sigma_{ij} = -p\delta_{ij} + \eta(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i})$ assuming an incompressible fluid (Note: define $p$).

**4)** (9 points; ~30%)

Consider a two-dimensional infinite laminar (parallel) linearly viscous flow in $x_1$ direction. In depth direction the flow is confined between the depths $x_2=0$ and $x_2=h$.

The viscosity $\eta$ is constant.

The flow velocity field is given as $\vec{v}(x_1, x_2) = \begin{pmatrix} \frac{\alpha}{\eta}x_2^2 + x_2 \\ 0 \end{pmatrix}$. The pressure in the fluid is given by $p(x_1, x_2) = \rho g x_2 + \alpha x_1$ and the body force is given by $F_1 = 0, F_2 = \rho g$

**a)** (2) Determine the strain-rate tensor at $(x_1, x_2)$

**b)** (2) Determine the stress tensor at $(x_1, x_2)$

**c)** (2) Determine the traction which the flow exerts ("Dutch": uitoefent) on the upper plate

**d)** (3) Show that the flow field is a solution of the Navier-Stokes equation.