$\bar{\sigma}$ ~~$\bar{\sigma}$~~ $= \bar{\bar{\sigma}} \bar{n}$ $\quad -\frac{1}{2}\sqrt{3}$

cos 150 =

$\bar{n} =$

Tentamen Continuummechanica (12-12-2014; docent: Spakman)

Write clearly and provide arguments in derivations and answers. Point-scores are given (40 in total).

Useful equations which you may, or may not need: $\frac{d\rho}{dt} + \rho\nabla.\vec{v} = 0$ | $\rho\frac{dv_i}{dt} = \frac{\partial\sigma_{ij}}{\partial x_j} + F_i$ | $\sigma_{ij}n_j = \sigma_i^{\hat{n}}$

$\frac{d}{dt} = \frac{\partial}{\partial t} + v_k\frac{\partial}{\partial x_k}$, $\sigma_{ij} = 2\mu\epsilon_{ij} + \lambda\theta\delta_{ij}$ | $\sigma_{ij} = -p\delta_{ij} + 2\eta\dot{\epsilon}_{ij} + \upsilon\dot{\theta}\delta_{ij}$ | $\rho\frac{dv_i}{dt} = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j \partial x_j} + F_i$

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1) (5) Prove the following identities using index notation:

a) (2) $\nabla\cdot(\phi\mathbf{v}) = \phi(\nabla\cdot\mathbf{v}) + \mathbf{v}\cdot\nabla\phi$

b) (3) $\nabla\times(\nabla\times\mathbf{u}) = \nabla(\nabla\cdot\mathbf{u}) - \nabla^2\mathbf{u}$ (Hint: $\varepsilon_{imn}\varepsilon_{ljk} = \delta_{mj}\delta_{nk} - \delta_{mk}\delta_{nj}$)

2) (4) Give the general definition of the stress tensor in an arbitrary point of a continuous medium.

3) (4) Derive the equation of mechanical equilibrium

4) (4) Determine the velocity gradient tensor, the strain rate tensor, and the rotation rate tensor for the following velocity field: $\vec{v}(\vec{r}) = \vec{\omega}\times\vec{r}$, where $\vec{\omega} = [0, \ 0, \ \Omega]^T$ is the angular velocity vector, Ω is the angular speed in rad/s, and $\vec{r} = [x_1, \ x_2, \ 0]^T$ is the position vector.

5) (11) Tractions [MPa] are given on the faces of a tiny square, which is rotated counter-clockwise over 150 degrees as indicated in the figure.

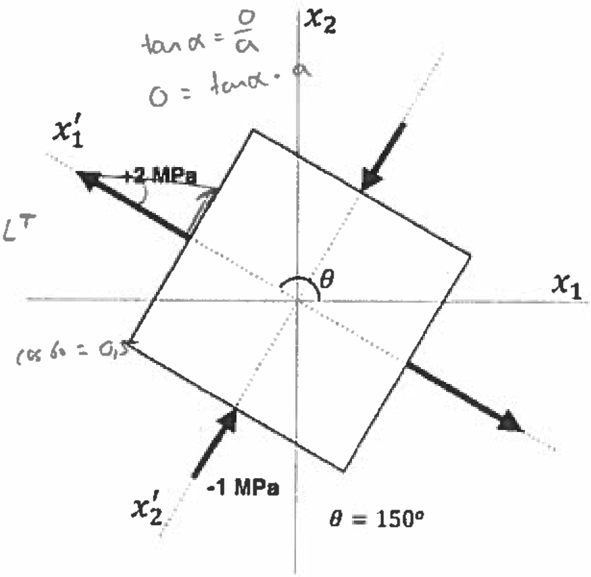
a) (2) Determine the stress tensor in the rotated frame. (explain your answer!)

b) (3) Determine the stress tensor in the $x_1 - x_2$ frame $\bar{\bar{\sigma}} = L\bar{\bar{\sigma}}' L^T$

c) (2) Which three attributes of the stress tensor can you use as an initial check on the correctness of the answer of b)?

d) (2) Determine the traction acting on a surface through the origin of the $x_1 - x_2$ frame with a normal pointing 60 degrees measured counterclockwise with respect to the x_1-axis.

e) (2) Determine the nature of the traction of d) [shear, normal, or other] Explain your answer.



6) (7)

Assume that the following stress field exists in a x_1-x_2-x_3 -frame: $\bar{\bar{\sigma}} = \begin{bmatrix} -x_3 & x_1x_2 & 0 \\ x_1x_2 & -x_3 & 0 \\ 0 & 0 & -x_3 \end{bmatrix}$

a) (2) Assuming mechanical equilibrium, determine the body force.

b) (2) Determine the principal stresses in P(1,2,3)

c) (3) Determine the principal axes in P(1,2,3)

7) (5)

Derive the Navier-Stokes equation from the General Equation of Motion using an incompressible linearly viscous fluid with constant viscosity η: *(provide explanation on all steps of the derivation)*

$$\rho\left(\frac{\partial v_i}{\partial t} + v_k\frac{\partial v_i}{\partial x_k}\right) = -\frac{\partial p}{\partial x_i} + \eta\frac{\partial^2 v_i}{\partial x_j \partial x_j} + F_i$$