DIVA: End-term test 2015

Write your name and student number, and write READABLE.

(1) 1. Solve the separable 1st order differential equation

$(x+xy)y'+y=0$ with the boundary condition $y(1)=1$

(1) 2. We have the inhomogeneous 1st order differential equation

$dx+(x-e^{y})dy=0$

First, solve the homogeneous equation. *Hint: solve for x in terms of y.*

Then require $C \to C(y)$ and solve the inhomogeneous equation

3. The solution of an inhomogenous 2nd order LDE with constant coefficients $y''+py'+qy=r(x)$ (p,q constant) is $y(x)=y_c(x)+y_p(x)$

With $y_c(x)=c_1y_1(x)+c_2y_2(x)$ and $y_p(x)=u(x)y_1(x)+v(x)y_2(x)$

Under certain conditions it appears that:

$$u(x)=\int\frac{-y_2r(x)}{W(x)} \text{ en } v(x)=\int\frac{y_1r(x)}{W(x)} \text{ met } W(x)=y_1y_2'-y_1'y_2$$

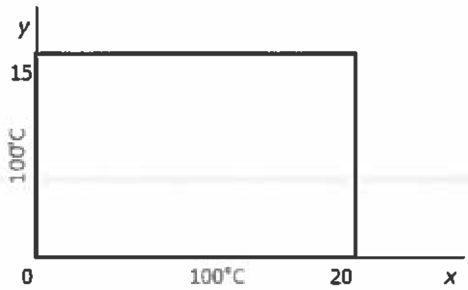
We can now solve any 2nd order LDE *(5 stappen plan)*

(1) a. $y''+16y=16\sin 4x$ (1) b. $2y''+y'=2x$

(2) 4. *Laplace:* $\nabla^2T=\frac{\partial^2T}{\partial x^2}+\frac{\partial^2T}{\partial y^2}=0$ with solutions $T=XY=\begin{Bmatrix}e^{ky}\\e^{-ky}\end{Bmatrix}\begin{Bmatrix}\sin kx\\\cos kx\end{Bmatrix}$

The steady state temperature distribution in a metal plate 10 cm square if one side (along x-axis) is held at 100°C and the other three sides at 0°C has a solution $T=\sum_{odd\ n}^{\infty}\frac{400}{n\pi\sinh n\pi}\sinh\frac{n\pi}{10}(10-y)\sin\frac{n\pi}{10}x$

Now consider a metal plate 15 x 20 cm square:



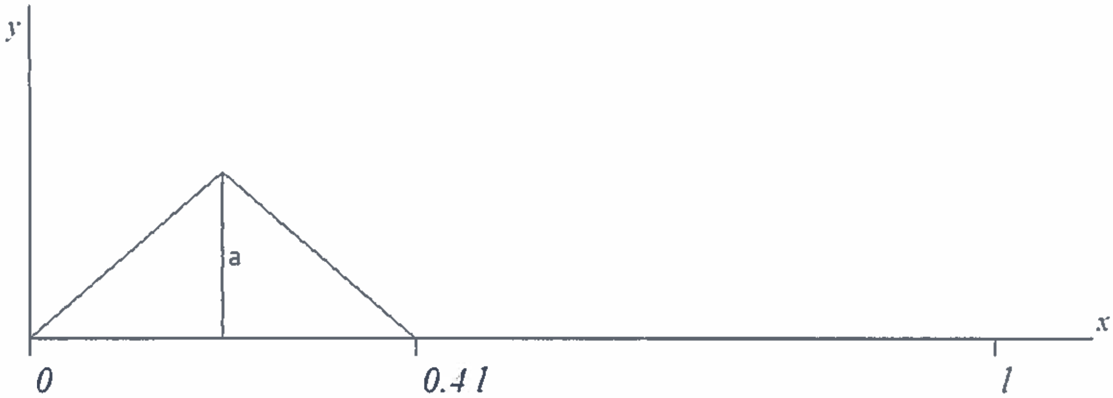
Find the steady state temperature distribution in this plate

(2) 5. *Diffusie equation:* $\nabla^2 u = \frac{1}{\alpha^2}\frac{\partial u}{\partial t}$ with solutions $u = FT = \begin{Bmatrix} e^{-k^2\alpha^2 t}\sin kx \\ e^{-k^2\alpha^2 t}\cos kx \end{Bmatrix}$

A bar of length l is initially at 0°C. From $t=0$ on, the $x=0$ end is held at T_1°C and the $x=l$ at T_2°C. Find the time-dependent temperature distribution.

(2) 6. *Wave equation:* $\nabla^2 y = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$ with solutions $y = XT = \begin{Bmatrix} \sin kx \\ \cos kx \end{Bmatrix}\begin{Bmatrix} \sin \omega t \\ \cos \omega t \end{Bmatrix}$

A string of length l has a zero initial velocity and a displacement as shown (plucked string problem)



Find the displacement as a function of *x* and *t*