GEO4-1415 Data processing and inverse theory

Tentamen - 8 Nov 2018 - 13h30-16h00 - BBG-079

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The numbers in () indicate percentage marks for the evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch, English, French, German or Luxembourgish.

1. (10) Consider a time signal of length T sampled with a rate $1/\tau$. The corresponding discrete Fourier Transform is defined on the domain $[-\nu_c, \nu_c]$ with a frequency step $d\nu$. What is the relation between τ, T, $d\nu$ and ν_c? Show that the time signal and the corresponding discrete Fourier Transform have the same number of points.

2. (10) Suppose you have a time signal $x(t)$. Give a schematic recipe to calculate the Hilbert Transform of $x(t)$. If you remember the definition of the Hilbert Transform that is of course a valid answer too.

3. (30) Consider to filters defined in the Z-domain:

$$H_1(z) = \frac{1-a}{1-az} \qquad\qquad H_2(z) = \frac{1-a}{1+az} \tag{1}$$

where $a = 0.9$.

Imagine you have an input wavelet $x(t)$ and you filter it with the impulse responses $h_i(t)$ to get an output $y(t)$. Noting that the filters defined in equation (1) are recursive, write these recursive relations for $y(t)$ in the time domain.

We now want to investigate which of the filters is high- or low-pass. To do this, fill in the table below, reasoning on the unit circle in the complex plane, and sketch the amplitude response of the filters.

Hint: use the general amplitude expression

$$\| H_i \| = \frac{1-a}{\sqrt{(1 \pm \alpha a)^2 + \beta^2 a^2)}} \tag{2}$$

for $z = \alpha + i\beta = e^{i\theta}$.

θ	0	$-\pi/4$	$-\pi/2$	$-3\pi/4$	$-\pi$
z					
ν					
H_1					
$\| H_1 \|$					
H_2					
$\| H_1 \|$					

4. (50) We want to design a causal filter $f_t = (a, b)$ which turns a wavelet $x_t = (1, 0, -1, 0, 1)$ into a wavelet $y_t = (0, 1, 0, -1, 0, 1)$.

Write the operation $x_t * f_t = y_t$ as a linear system $d = Gm$.

Solve this system using SVD. Don't forget to check that the singular value decompostion of G is correct.

Is the filter f_t you found optimal?

Looking at the filter you found in the Z-domain, does your result sound plausible?

Good luck.