GEO4-1415 Data processing and inverse theory

Tentamen - 15 Apr 2009

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The numbers in () indicate the percentage for evaluation. No documents are allowed during the examination. Please write clearly and feel free to write your answers in Dutch.

1. (30) Plot the wavelet $a_t = (0, 1, 2, 1, 0)$. Now consider the wavelet $b_t = (1, -1)$. Convolve $a_t * b_t$ and show that b_t is a finite difference operator. We now want to integrate a_t. We therefore construct the inverse wavelet of b_t. Do this using the Z-transform of b_t. Give the infinite expression of b_t. Convolve a_t with the inverse of b_t of length 2, 3, 4 etc. Can you find a better way of doing this?

2. (20) A low-pass Butterworth filter filter has the amplitude response

$$|A(\omega)|^2 = \frac{1}{1 + (\omega/\omega_c)^n} \tag{1}$$

   For $\omega_c = 2\pi$ find the amplitude at different ω for different n. How does the value of n affect the frequency content of the output above and below the cut-off frequency ω_c.

3. (50) We want to solve the system

$$x + y + z = 3 \tag{2}$$

$$x + y = 1 \tag{3}$$

   Evaluate the unknowns with a damped least-squares solution

$$m = (G^t G + \theta I)^{-1} G^t d = G^t (G G^T + \theta I)^{-1} d \tag{4}$$

   Solve the system for various values of $\theta = 1,\ 0.1,\ 0.01$. Explain what happens to the solution by looking at the resolution operator. Now solve the system using a singular value decomposition. For which θ are the solution from SVD and DLS equal? Why?

Good luck.