GEO4-1415 Data processing and inverse theory

Tentamen - 5 Nov 2015 - 13h30-16h30 - BBG-065

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The numbers in () indicate percentage marks for evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch or English.

1. (20) The discrete Fourier Transform is given by the expression

$$F_p = \sum_{n=0}^{N-1} f_n w^{-np} \tag{1}$$

   with $w = exp^{i2\pi/N}$ and $p = (0, 1, ..., N-1)$

   We give 2 causal wavelets $a_t = (2, -1, 0)$ and $b_t = (4, 1, 0)$. Calculate their convolution and the cross-correlation. Calculate the discrete Fourier Transform of the 2 wavelets using equation (1) of the first question. Show that the convolution theorem holds for the convolution of the 2 wavelets you found above.

2. (30) Suppose a seismic source below the base of the weathering layer. Ghost energy is reflected at the base of the weathering layer, where the reflection coefficient approaching from below is -R. An impulse followed by its ghost constitutes a filtering action. Ignoring multiple reflections, write the Z-transform of this filter knowing that the time delay of the ghost is the two-way travel-time τ from the source to the reflector. See Figure 1.

   As a numerical example, take a source located 8 m below the weathering layer. The velocity in the source layer is 2 km/s. A wavelet $y_t = (1, -0.3, -0.15, -0.075, -0.1)$, sampled every 4 milliseconds, is recorded at the surface. Knowing that the reflection coefficient R=0.8, find the source wavelet by polynomial division in the Z-domain.

   An alternative approach is to construct a recursive filter in the time domain. Design this filter and convolve the recorded signal from the ghost. You should find the same answer as above.

3. (50) We have a borehole equiped with 2 geophones to measure travel times between an explosion at the surface and the instruments. We know that the structure may be parameterized with 3 layers of 1 km thickness and corresponding velocities $v_1 = x, v_2 = y, v_3 = z$ as shown on Figure 2. The measurement of the 2 travel times yielded $t_1 = 3$ and $t_2 = 1$ seconds. We want to find the velocities in the layers. When you write the problem down in its matrix form, you will get the following system

$$x + y + z = 3 \tag{2}$$

$$x + y = 1 \tag{3}$$

Evaluate the unknowns with a damped least-squares solution

$$m = (G^tG + \theta I)^{-1}G^td = G^t(GG^T + \theta I)^{-1}d \tag{4}$$

Choosing either of the 2 formulations, solve the system for various values of $\theta = 1,\ 0.1,\ 0.01$ etc. Explain what happens to the solution by looking at the resolution operator. Now solve the system using a singular value decomposition. For which θ are the solution from SVD and DLS equal? Why?

Good luck.

Figure ①

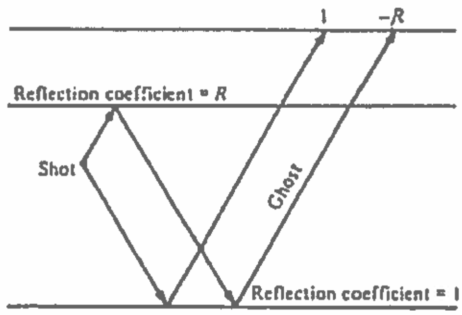

Figure ②