GEO4-1415 Data processing and inverse theory

Tentamen - 7 Nov 2013 - 13h30-16h30

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The numbers in () indicate the percentage for evaluation. No documents are allowed during the examination. Please write clearly and feel free to provide your answers in Dutch or English.

1. (20)
   - Give the mathematical definition of convolution and cross-correlation and explain why they are important in signal processing.
   - State the sampling theorem.
   - A time series of length T is sampled with an time interval τ. Define the Nyquist frequency and give the sampling interval in the frequency domain.
   - A filter has the Z-transform $F(Z) = \frac{a_0+a_1z+a_2z^2}{b_0+b_1z}$. For a given input wavelet X(Z), the signal output Y(Z)=F(Z)X(Z). Give the corresponding expression for y_t in the time domain.

2. (30) We propose to predict the prime numbers with a simple linear prediction filter! The wavelet x composed of the first 5 prime numbers is (2, 3, 5, 7, 11). Construct a linear prediction filter (f_0, f_1) of prediction length 1 which predicts the next prime number. Write the system of equation using the auto-correlation of the input wavelet. Once you found the filter coefficient you can check how well it works by evaluating $(c_1, ..., c_6)$=x*f. The values $c_1 - c_5$ correspond to the prediction of the input wavelet. Does it work? c_6 is the prediction of the next prime number which is 13. Does is work? Explain!

3. (50) Consider the following system of simultaneous equations:

$$\begin{aligned} x - y - 2z &= 6 \\ 3x + y + z &= 11 \end{aligned}$$

   Solve this system $d = Gm$ by singular value decomposition. Is the solution equivalent to a least-squares solution or a minimum norm solution. Why? Check your answer. We call this solution $\tilde{m}$ Now we make a model transformation

$$\begin{aligned} x' &= x \\ y' &= y/2 \\ z' &= z/2 \end{aligned}$$

   Write the new system of equations d=G'm' where m'=Sm, where S is a specific transformation matrix. Now solve this new system with the general minimum norm solution $\tilde{m}' = C_{m'}G'^t(G'C_{m'}G'^t)^{-1}d$. What should $C_{m'}$ be for $\tilde{m}'$ to be equivalent to $\tilde{m}$

Good luck.