GEO4-1415 Data processing and inverse theory

Tentamen - 6 Nov 2014 - 13h30-16h30

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The numbers in () indicate the percentage for evaluation. No documents are allowed during the examination. Please write clearly and feel free to give your answers in Dutch or English.

1. (10)

   - A continuous time series is given by the expression $cos[2\pi(12.5)t]$. This time series is digitized with a sampling interval of 0.03 seconds. Does this give a correct representation of the original time series? Why? Same question for a sampling interval of 0.05 seconds.
   - Give the expression for the impulse response of 2 filters in parallel and 2 filters in series.

2. (40) We define a digital filter $d_n = (1, -1)$. In a first approximation this filter acts as a derivative operator. You can see this by applying this filter to a wavelet $a_n = (0, 1, 2, 3)$. Make a drawing of a and $b = a * d$ and explain why d is called the derivative operator.

   We now want to find the inverse of the derivative operator which is the integral operator i. Find the optimal integral operator of length 2 so that $i * d = d * i = \delta$. Apply the integral operator to b, which you found above, and plot it on the same graph. Do you find a back? Explain.

   Rather than designing a Wiener integral filter, you can construct a recursive filter to undo the effect of the derivative filter. Write therefore an expression in the time domain for the operation $C(z) = \frac{1}{1-z}B(z)$ and evaluate c_n. Compare c_n to a_n. Explain what you find and why.

3. (50) We will now come back to Wiener filter from the previous question. Write the problem $d * i = \delta$ as a system of linear equation $Gx = y$, where y is the vector representing δ and x the vector representing the 2-dimensional integral operator to be found. G is a 3x2 matrix.

   Solve this system by singular value composition. Hopefully you'll find the same answer as before. Why?

   Give the data and model resolution.

Good luck.