

GEO4-1415 Data processing and inverse theory

Hertentamen - 31 May 2007

Jeannot Trampert

The numbers in () indicate the percentage for evaluation. No documents are allowed during the examination. Please write clearly and feel free to write your answers in Dutch.

1. (20) The discrete Fourier Transform is given by the expression

$$F_p = \sum_{n=0}^{N-1} f_n w^{-np} \quad (1)$$

with $w = \exp^{i2\pi/N}$ and $p = (0, 1, \dots, N-1)$

Show how the mean of the signal f_n is related to F_0 . Calculate the Fourier transform of $f_n = (0, 1, -1, 0)$

2. (20) Suppose that we have a wavelet $a_t = (a_0, a_1, \dots, a_N)$. Give the corresponding Z-transform.

Show that the convolution of 2 wavelets in the time domain corresponds to a multiplication of the respective Z-transforms.

Suppose that the output of a filter in the Z-domain is given by $Y(Z) = F(Z).X(Z)$, where $X(Z)$ is the input, $Y(Z)$ is the output and $F(Z) = \frac{a_0 + a_1 z + a_2 z^2}{1 + b_1 z}$ is the filter. Write the corresponding expression for y_t in the time domain.

3. (10) State

- The sampling theorem
- Parseval's theorem

4. (50) We have a borehole equipped with 2 geophones to measure travel times between an explosion at the surface and the instruments. We know that the structure may be parameterized with 3 layers of 1 km thickness and corresponding velocities v_1, v_2, v_3 as shown on the figure. The measurement of the 2 travel times yielded $t_1 = 1$ and $t_2 = 3$ seconds. We want to find the velocities in the layers. The problem is easiest to solve if we use the slowness $s_i = 1/v_i$ giving the system $t = Gs$ where

$$G = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$$

First solve the problem using singular value decomposition. Give the solution and the corresponding resolution matrix. Make an interpretation of the resolution matrix you found.

Now solve the problem using the minimum norm solution. Compare the 2 solutions and explain why they are different or similar.

Good luck.

Bo Charles
c.m.a. hof

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