Exam Principles of Groundwater Flow

November 8, 2018

Succes!

1. A fully penetrating drinking water supply well, with a well diameter $d_w = 500$ mm, is located in a confined aquifer, with thickness $D = 75$ m. The volumetric flow rate is $Q = 300$ m^3/hour. The hydraulic conductivity of the aquifer is $k = 10$ m/day[1] and its porosity is equal to $n = 0.2$. At a distance $r = 200$ m from the well, the hydraulic head is constant: $h(200) = 150$ m. Note that this constant head boundary is a concentric circle, with the well in its midpoint.

   (a) Determine the hydraulic head at a distance $r = 100$ m , measured from the center point of the well.

   (b) The general expression for the travel time of a particle is given by

   $$t_{\text{travel}} = \int_{r_1}^{r_2} \frac{1}{v(r)}\, dr, \qquad (1)$$

   where $v(r)$ is the effective groundwater velocity. Give a formula for the travel time in terms of the relevant parameter of this problem.

   (c) Determine the travel time of a particle that is released at a distance $r = 100$ m towards the pumping well.

   (d) An important issue with respect to protection of drinking water areas is the so-called 25-year protection zone. This defines a cir-

[1]Note: Be aware of the different UNITS!!! mm, m, years,days, etc.

cle around a pumping well in which certain activities that can affect the groundwater quality are forbidden, e.g. waste disposal, fertilization of the soil, etc. If the radius of the pumping well is assumed to be small compared to the other spatial dimensions[2], show that the time needed to travel from a location R towards the well is given by:

$$t(R) = \frac{\pi n D R^2}{Q} \tag{2}$$

(e) Determine the radius of the 25-year protection zone for the aforementioned water supply well.

(f) Next we consider the same aquifer, but now with constant recharge (infiltration) through the upper confining clay layer. The recharge is given by N [m/day]. Continuity requires that

$$Q = -2\pi D r q(r) + \pi r^2 N \tag{3}$$

Explain in words the different terms in this equality and explain the signs of all terms

(g) Derive an expression for the travel time using the definition given by (1) and expression (3). (Hint: determine $q(r), v(r)$, the definition of travel time (1), and integrate from $r_w = 0$ to $r = R_{25}$)

(h) If we recast the formula found under question (g) in terms of the 25-year protection zone we obtain

$$R_{25}^2 = \frac{Q}{\pi N}\left(1 - e^{-\frac{N}{nD}t_{25}}\right) \tag{4}$$

Here, t_{25}=25 years and R_{25}= the radius of the 25-year protection zone with uniform recharge N. Compute the radius of the protection zone if $N = 0.25$ m/year. Compare the result with the result found under (e), i.e. where N=0 m/year.

2. Indicate whether the following statements are true or false. Motivate your answers.

[2]Implying $r_w = 0$

(a) The porosity of a water-saturated porous media can be dermined from the wet bulk density ρ_b^w, the fluid density ρ_f and the solid density of the soil particles ρ_s according to

$$n = \frac{\rho_s - \rho_b^w}{\rho_s - \rho_f}$$

(b) The mass balance equation for a fluid in a deformable porous medium is given by

$$n\frac{\partial \rho}{\partial t} + \rho\frac{\partial n}{\partial t} + \text{div}\,(\rho\bar{q}) = 0$$

(c) According to Karl von Terzaghi's Law $\sigma_{vt} = P + \sigma_{ve}$, subsidence is a completely reversible process.

(d) In three-dimensional anisotropic aquifers the specific discharge vector is never perpendicular to the constant head contours.

(e) The Dupuit approximation implies that the vertical pressure in a confined aquifer is constant

(f) Heterogeneous aquifers are always anisotropic.

(g) The one-dimensional hydraulic head distribution in both a confined and unconfined aquifer is **independent** of the hydraulic conductivity, provided there is **no** recharge

3. A transient pumping test is conducted in order to determine the transmissivity T m^2/s and storativity S [-] of a confined aquifer. The pumping rate is 10 m^3/hour. The thickness of the aquifer is $D = 50$ m, and the porosity is $n = 0.4$. Initially, the hydraulic head distribution is horizontal, and given by $h(x, y, 0) = h_0(x, y) = 60$ m. Two observation wells are installed: one well (A) at 10 m and one well (B) at 20 m from the well. At a certain time instant after the pumping started the respective hydraulic heads observed in the wells are: $h_A = 55$ m and $h_B = 57$ m. Assume that the Theis solution can be approximated by the truncated series expansion for the well function.

   (a) Determine the transmissivity T of the aquifer and the hydraulic conductivity.

(b) The drawdown in observation wells A and B are observed after $t = 2.7183$ days. Determine (compute) the storativity S of the aquifer

(c) The truncated series expansion for the well function $W(u)$ gives a good approximation if $u << 1$. Let's assume that u has to be less than 0.01, i.e $u < 0.01$. Is this condition satisfied for both observation wells?

(d) Theis solution for a transient pumping well is derived assuming a variety of conditions. List four of these conditions.

(e) If the transmissivity decreases to half of its original value, and all other above mentioned observations/conditions remain the same, what would be the consequence for the storativity value S?

4. Consider a confined aquifer with length L [m] and thickness D [m]. At $x = 0$ [m], the two-dimesional volumetric flow rate Q'(0) is given, while at $x = L$ [m], a constant head boundary condition exists: $h(L) = h_L$ [m] (surface water, e.g. a ditch or canal). The aquifer consists of two parts with different hydraulic conductivity: for $0 < x < \frac{L}{2}$ the conductivity is k_1 [m/s] (we will refer to this region as region 1) and for $\frac{L}{2} < x < L$ the conductivity is k_2 [m/s] (to which we refer as region 2).

   (a) What will be the expected shape of the hydraulic head distribution $h = h(x)$ in both regions?

   (b) Both regions are conneceted at $x = \frac{L}{2}$. If $k_1 > k_2$, make a sketch (drawing) of the head distribution in the vicinity of $x = \frac{L}{2}$. EXPLAIN why you think this is the correct picture'.

   (c) Determine $Q'(L)$.

   (d) We assume that the hydraulic head at $x = \frac{L}{2}$ is given by $h(\frac{L}{2}) = h_m$, where h_m is yet unknown. Moreover the head at the constant flow boundary at $x = 0$ is unknown, i.e. $h(0) = h_0$. Determine both h_m and h_0. Hint: do NOT use the differential equations but use the discrete expressions for Q' in both regions. This is possible because of the correct answer to question a).

   (e) **Bonus question** See the next page.

Show (derive!) that the head distribution in the aquifer is given by

$$\text{for } 0 > x > \frac{L}{2} \rightarrow h_1(x) = -\frac{Q'}{k_1 D}x + \frac{Q'L}{2D}\left[\frac{1}{k_1} + \frac{1}{k_2}\right] \tag{5}$$

$$\text{and for } \frac{L}{2} > x > L \rightarrow h_2(x) = -\frac{Q'}{k_2 D}[x - L] + h_L \tag{6}$$

..... The end .....