Geo3-1313

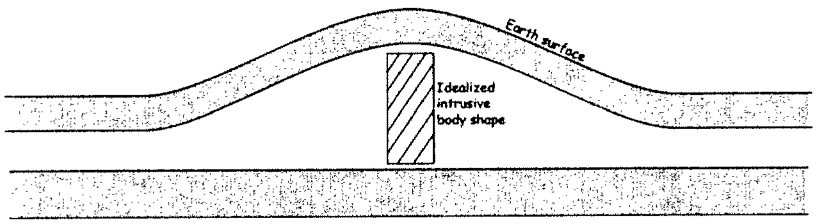
Examination "Geodynamics" April 19, 2010

Write your answers to the questions of the three parts of this exam (1: Govers, 2: Wortel, 3: v.d.Berg) on separate sheets of paper.

In engineering, the flexure equation that is used to compute the bending of 2D, thin elastic sheets is

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(D\frac{\mathrm{d}^2 w}{\mathrm{d}x^2}\right) = q + P\frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \tag{1}$$

(a) We wish to investigate flexure due to a *laccolith*, which is an igneous intrusive body. The pressure in the intrusive body is high enough to lift the surface crust layer (see figure below). Carefully argue and show how (1) needs to be modified to make it applicable to the bending of a shallow crustal layer following laccolith emplacement beneath it. You may assume that the intermediate (white) crustal layer deforms/flows very easily.



Idealized geometry of crustal layers (white and grey) following intrusion of a laccolith in between them. The intrusive body has density ρ_i, and crustal layers have density ρ_c.

(b) The Green function of the above problem *(D* uniform, $x \geq 0$, *P=0)* reads

$$G(x) = G_0 e^{-kx} \cos kx$$

Derive an expression for the maximum horizontal tensile stress directly above the laccolith, assuming a thickness *H*, Young's modulus *E*, and Poisson's ratio ν.

(c) Why is it natural to express rheological properties in terms of invariants of the involved tensors (stress, strain rate, ...)? Can all rheological properties be written this way (arguments please)?

Opgave 2: In search of the isostatic compensation mechanism

Part 1: Gravity field

Consider the situation in which a marine geophysical investigation of a submarine ridge type of structure [not a spreading (mid-)oceanic ridge, but a ridge resembling the Hawaiian Swell] has been carried out. As a result gravity field data, as well as bathymetry data, are available for the entire region of interest. The aim of the study is to gain insight in the mechanism by which the load (of the ridge) is compensated.

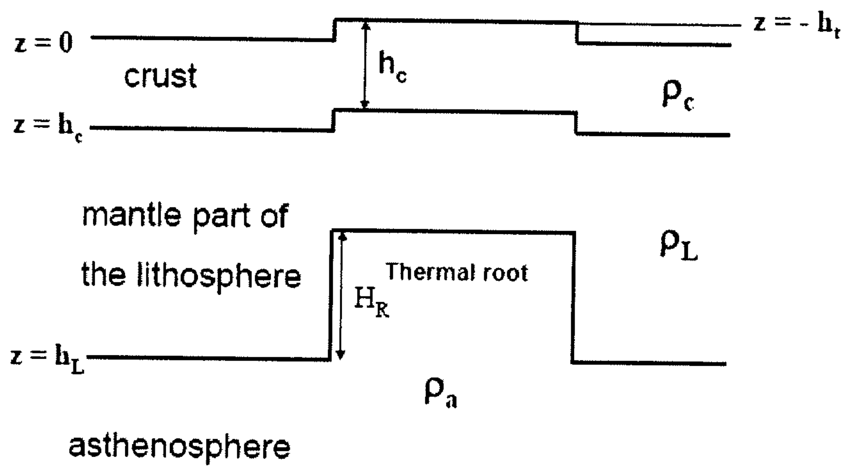
Describe briefly the methodology using "special functions " (such as the gravitational admittance) in the analysis of the gravity field and bathymetry data by which you would try to achieve this aim.

Part 2: Geoid

Consider a plateau surrounded by a reference continental type of lithosphere (data concerning reference lithosphere: surface at sealevel; crustal thickness is 35 km, crustal density ρ_c = 2850 kg/m^3; total thickness of the lithosphere, including crust, is 135 km). The elevation of the plateau relative to the surface of the reference continental lithosphere is h_t. The subject of investigation is the deep structure compensating the topography of the plateau. Two mechanisms (or structures) are considered :

(1) Airy type of isostasy pertaining to the crust (only a crustal root), and

(2) A thermal root (that means: crustal thickness unchanged, but with thinning of the mantle part of the lithosphere). See figure below for the thermal root geometry.



For both mechanisms local isostatic compensation may be considered to be valid. Make reasonable assumptions on the densities of the asthenosphere and the mantle part of the lithosphere.

a. Explain – by physical arguments - why in particular a study of the regional geoid anomalies may provide the possibility to discriminate between a thermal root and an Airy-type of isostatic compensation (in other words: explain the special merits of studying the geoid in this case).

For the thermal root structure only:

b. Derive an expression for the geoid anomaly (relative to the reference situation), immediately after the instantaneous formation of the structure at t = 0 (that is the situation shown in the figure).

c. Consider the case in which the elevation of the plateau h_t = 800 m. Now calculate H_R, and the predicted geoid anomaly (relative to the reference situation) for the thermal root mechanism. Gravitational constant $G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}. Give a qualitative description of the evolution of the anomaly with time (ignore erosion in this answer).

d. Is there a maximum to the height of a plateau which can be explained by the thermal root mechanism? If there is: Explain why. If there is not: Explain why not.

Note: For a useful comparison or test the anomaly of the other Airy type of structure (#1) should be quantified as well; this is not part of this "opgave").

Part 3: Physics of the earth's interior

Problem 1 Consider the gravity field of a homogeneous spherical planet with a uniform density $\rho = \rho_0$, outer radius $r = R$ and total mass M. The gravity potential U for this spherically symmetric model is a solution of the Poisson equation,

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dU}{dr}\right) = 4\pi G\rho(r) \tag{1}$$

1. Derive the following expressions for the internal gravity acceleration g_{int} and gravity potential U_{int}, by solving (1),

$$U_{int}(r) = \frac{2\pi}{3}G\rho_0 r^2 - \frac{3}{2}\frac{GM}{R}, \quad g_{int}(r) = \frac{4\pi}{3}G\rho_0 r \tag{2}$$

   Apply the boundary conditions $lim_{r\to\infty}U(r) = 0$, $g(0) < \infty$ and continuity of U and g at $r = R$.

2. Consider the following thought experiment where a small (point)mass object falls through a narrow hole drilled, in a straight line, from the surface of the uniform density sphere, through the centre to the antipodal point at the surface.

   Assume the motion of the point mass is without friction and with zero initial velocity. Derive the following expression for the trajectory (time function of the spatial coordinate) of the oscillating motion,

$$r(t) = R\cos\left(\sqrt{\frac{4\pi}{3}G\rho_0}\, t\right) \tag{3}$$

   Note: in (3) r varies between R and $-R$ and changes sign at the centre of the sphere.

   Hint: apply Newtons second law, $F = ma$, equating the gravity force on the point mass to the product of its mass and acceleration.

3. Compute the period of the oscillation for 'Earth-like' parameters, using $G = 6.67 \times 10^{-11} Nm^2kg^{-2}$.

Problem 2

1. Derive the Williamson-Adams (W-A) equation for the density distribution of an adiabatic planetary mantle,

$$\frac{d\rho}{dr} = -\frac{\rho^2 g}{K} \tag{4}$$

2. Discuss the numerical solution of (4) by Williamson and Adams where they applied available radial distributions of the seismic wave velocities v_p and v_s.

   Hint: use the following relations between the seismic wave velocities and the elastic parameters, $v_p = \sqrt{(\lambda + 2\mu)/\rho}$, $v_s = \sqrt{\mu/\rho}$ and $K = \lambda + (2/3)\mu$.

3. Discuss the relevance of the assumption of adiabatic conditions when applying the W-A equation (4) to the Earth's mantle.
4. What did Williamson and Adams learn from their computed density profile for the Earth's mantle?
5. Discuss the extension of the above W-A investigation by Bullen, who also used the moment of inertia to draw conclusions on the density distribution of the mantle.