Examination "Geodynamics" April 18, 2012

Write your answers to the questions of the three parts of this exam (1: v.d.Berg, 2: Govers, 3: Wortel) on separate sheets of paper.

12/04/18

Part 1: Physics of the earth's interior

Problem 1

From measurements of the surface heatflow density the present day heat output of the Earth is estimated to be $Q_s = 44 \cdot 10^{12}$ W.

- 1. Discuss different heat sources that contribute to the present heatflux Q_s and indicate their relative importance
- 2. Investigate a steady state model for the temperature distribution of the Earths interior. Assume a spherically symmetric and homogeneous model (without a distinct core) where the temperature and the material properties depend only on the radial coordinate r. Assume the following model parameter values: $R=6\cdot 10^6$ m the outer radius, $\rho=5\cdot 10^3$ kgm⁻³ the density, $k=5 \text{WK}^{-1}\text{m}^{-1}$ the thermal conductivity and $H=5\cdot 10^{-12} \text{Wkg}^{-1}$ the density of the internal heat production rate.

The steady state temperature distribution is described by a Poisson equation,

$$k\nabla^2 T + \rho H = 0 \tag{1}$$

Solve equation (1) ¹ for a given surface temperature $T(R) = T_s = {}^{0}C$ and evaluate the temperature in the centre r = 0 and halfway between centre and the surface.

- 3. How does the surface heatflux of this model relate to the observed flux Q_s mentioned above?
- 4. Discuss the shortcomings of the above model and give a description of a more complete physical model for the Earth's internal temperature that takes into account large scale flow processes manifested by plate tectonics.

Problem 2

Minerals in the earth's mantle are subject to solid state phase transitions and the the crystallization of the solid inner core is another example of phase transitions in the Earth's interior.

- 1. Discuss examples of phase transitions that have been used to determine the temperature in the Earth's interior.
- 2. Discuss the different rol of exothermal and endothermal phase transitions on the dynamics of the Earth's mantle.

Substitute for the Laplace operator $\nabla^2 T = \frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dT}{dr} \right)$, applying spherical symmetry.

Question 1

One of the flexure equations we derived was

$$\frac{\mathrm{d}^2}{\mathrm{d}x^2} \left(D(x) \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} \right) + (\rho_a - \rho_i) gw - P \frac{\mathrm{d}^2 w}{\mathrm{d}x^2} = q(x)$$
 (1)

- a) (3 pt) Give the meaning, S.I. unit, and typical value in the context of lithospheric flexure of each of the symbols in equation (1).
- b) (3 pt) What physical assumptions were used in the derivation of equation (1)? Under which conditions are these assumptions warranted?
- c) (4 pt) Determine the flexural response due to two line loads, at x=a and x=3a, using the symmetric Green function for a uniform plate and P=0;

$$w(x) = \frac{k e^{-kx}}{2g(\rho_a - \rho_i)} (\cos kx + \sin kx) \quad (x \ge 0), \qquad k \equiv \left(\frac{(\rho_a - \rho_i)g}{4D}\right)^{1/4}$$

Deel R. Wortel

Opgave 1:

Consider the situation in which a marine geophysical investigation of a region has been carried out. As a result gravity field data, as well as bathymetry data, are available for the entire region of interest. The aim of the study is to gain insight in the mechanism by which topographic features are compensated.

- a. Describe briefly the use and merits of the use of "special functions" (such as the gravitational admittance) in the analysis of the gravity field and bathymetry data by which you would try to achieve this aim.
- b. Derive the gravitational admittance Z(k) for the case of Airy-compensated topography (see Figure 1)

Airy (Compensated Topography)

$h(x) \qquad \text{Water} \qquad d$ Oceanic Crust p_c Mantle

r(x)

Figure 1: Periodic variation in surface topography at depth and Moho topography.

Oceanic crust is indicated in grey.

 ρ_m

Opgave 2:

Consider a region with lateral variations in crustal and lithospheric structure. Explain why geoid anomalies in such a region contain information concerning the stress field in the lithosphere (including the crust).

N.B.: The explanation should contain the physical basis for the relationship between geoid anomalies and stress field.