Land Surface Process Modelling, exam, Thursday, April 2nd, 2015, 1.30-4.30 pm

Please give your answers in Dutch or English. Each sub-question (2a, 3b, etc.) will approximately have the same weight in calculating your mark.

Question 1: definitions

a) What is a probability function or probability distribution function? Also, give an example.

b) What is the difference between a conceptual and a physically-based model? You can give an example of (a) model(s) to illustrate your answer.

Question 2: Python programming

```
names = ['piet', 'klaas', 'jan', 'paul']

i = 0
while i < len(names):
  print names[i]
  i = i+1
```

Table 2.1 A Python program.

Have a look at the Python program in Table 2.1.

a) What does the program in Table 2.1 print?

b) Rewrite the program in Table 2.1 by encapsulating the while loop in a function. Your new program should print the same!

c) Modify the program in Table 2.1 in such a way that it prints of each name (in the list `names`) the second letter.

d) Modify the program in Table 2.1 in such a way that it still reads the names from the list, but printing for each letter 'l' in a name a capital 'L'. Do not use the string module. The program should print:

```
p i e t
k L a a s
j a n
p a u L
```

e) Rewrite the program in Table 2.1 by replacing the `while` loop with a `for` statement. Your new program should print the same as the one in Table 2.1!

Question 3: calibration and sensitivity analysis

To predict total runoff from a a small catchment as a result of a single rainstorm, a simple, lumped, static, rainfall-runoff model is used:

$$q = \max(ap - b, 0) \qquad (3.1)$$

with:

p	total rainfall in the catchment (m^3)
a	fraction of rainfall reaching the soil (-)
b	infiltration capacity of the catchment (m^3)
q	total discharge at the outflow point of the catchment (m^3)

Note that max(x,y) in 3.1 assigns the maximum value of x and y, i.e. q cannot get less than 0. Note that a and b are parameters, p is an input variable and q is the output variable of the model. It is known that a ranges between 0 and 0.4 while b ranges between 0 and 40.

To calibrate the model, data from one rainstorm are available. The total rainfall during that rainstorm was 100 m^3, the total discharge was 30 m^3.

a) Provide a goal function that could be used to calibrate the model using these data (give the equation that you would use as goal function).

b) Sketch the response surface for the parameters a and b with the goal function you provided in 2a). Use the values in Appendix 1. Do not forget to provide axis annotation. You can use the squared paper ('ruitjespapier') provided.

Question 4: numerical solution of differential equations

The level of a lake changes as a result of net input to the lake (rain plus inflow minus evaporation) and output from the lake through a pipe one metre above the lake bottom. The change in the level z (m) of the lake is described by the equation:

$$\frac{dz}{dt} = r - k(z-1) \qquad (4.1)$$

with: z, the depth (m) of the water in the lake; t, the time (years); r, the net input (m/year) to the lake; k, a constant (year^{-1}) whose value is related to the geometry of the pipe. Note that z is always greater than 1. Equation 4.1 can be solved numerically using the Euler method, the more complicated Heun method or the Runge-Kutta method.

k	0.01 year^{-1}
r	0.5 m/year

Table 4.1. Values of k and r in equation 4.1.

Values of k and r (both assumed to be constant) are given in Table 4.1. In 1980, the depth (z) of the lake was 18 m.

a) Calculate the depth (z) of the lake in 1982 using the Euler method. Use a time step of 2 years.

Table 4.2 provides the Runge-Kutta algorithm.

b) Calculate the depth (z) of the lake in 1982 using the Runge-Kutta method. Use a timestep of 2 years.

ALGORITHM RUNGE–KUTTA (f, x_0, y_0, h, N).

This algorithm computes the solution of the initial value problem $y' = f(x, y)$, $y(x_0) = y_0$ at equidistant points

$$x_1 = x_0 + h,\ x_2 = x_0 + 2h, \cdots, x_N = x_0 + Nh;$$

here f is such that this problem has a unique solution on the interval $[x_0, x_N]$ (see Sec. 1.9).

INPUT: Initial values x_0, y_0, step size h, number of steps N

OUTPUT: Approximation y_{n+1} to the solution $y(x_{n+1})$ at $x_{n+1} = x_0 + (n + 1)h$, where $n = 0, 1, \cdots, N - 1$

For $n = 0, 1, \cdots, N - 1$ do:

$k_1 = hf(x_n, y_n)$

$k_2 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_1)$

$k_3 = hf(x_n + \frac{1}{2}h, y_n + \frac{1}{2}k_2)$

$k_4 = hf(x_n + h, y_n + k_3)$

$x_{n+1} = x_n + h$

$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$

OUTPUT x_{n+1}, y_{n+1}

End

Stop

End RUNGE–KUTTA

Table 4.2. Runge-Kutta algorithm (taken from Kreyszig).

Question 5: modelling approaches

As a result of reduced flooding, floodplains that used to have a grass vegetation cover may change into areas containing predominantly Cambara trees (Figure 5.1). A model is needed that simulates invasion of a grassland area by Cambara vegetation. Cambara spreads mainly as a result of seed transport over relatively short distances up to 100 m.

a) Assume the model is built as a cellular automata model. Give the model structure of such a model. Provide state variables that could be used, and a transition function simulating changes over a time step. You do not need to give details, give the general approach that could be followed.

b) Another approach would be to use an agent-based model for this purpose. Give the model structure of such a model. Include a short explanation of model component(s) represented by agents and rules needed to describe agent behaviour.

c) Given that you have the choice, which approach (cellular automata model or agent-based model) would you choose to model the invasion of grassland? Argument why you favour one over the other.

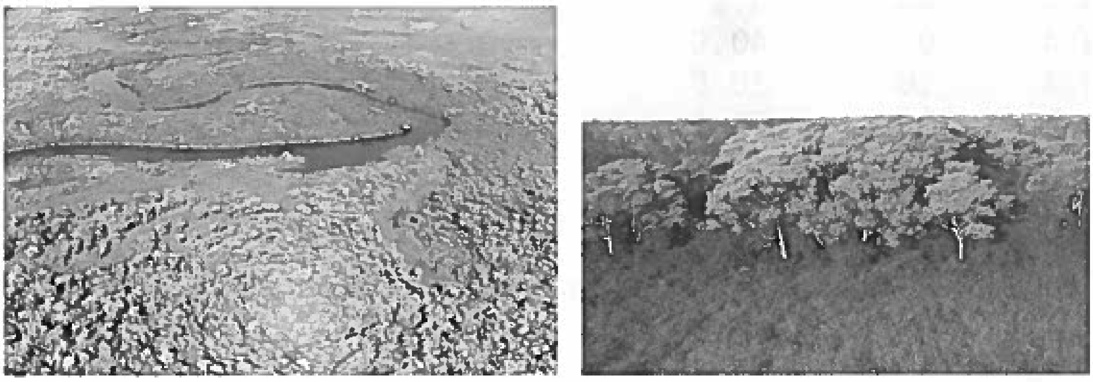

Figure 5.1 Cambara trees.

Appendix 1

a	b	q
0.0	0	0.0
0.0	10	0.0
0.0	20	0.0
0.0	30	0.0
0.0	40	0.0
0.1	0	10.0
0.1	10	0.0
0.1	20	0.0
0.1	30	0.0
0.1	40	0.0
0.2	0	20.0
0.2	10	10.0
0.2	20	0.0
0.2	30	0.0
0.2	40	0.0
0.3	0	30.0
0.3	10	20.0
0.3	20	10.0
0.3	30	0.0
0.3	40	0.0
0.4	0	40.0
0.4	10	30.0
0.4	20	20.0
0.4	30	10.0
0.4	40	0.0

Values of q (m^3) for different values of a (-) and b (m^3) in the model given by equation 1.1 for $p = 100$ m^3.