Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

Part I (Spiers) 12-03-2009 14.00 -17.00 hours Minnaert 019

Note:

- The duration of this exam is 3 hours.
- Answer any 4 of the 7 questions given.
- All questions count with equal weight to the final grade.
- Allow about 45 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you identify all mathematical symbols used in answering the questions (marks will be deducted for unidentified symbols).
- Use SI units unless otherwise specified.

Good luck all !!!

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a) Using MATRIX NOTATION, write down the stress-strain relations for an anisotropic elastic material.

b) Taking into account the symmetry of the stiffness matrix (C_{rs}), the non-zero components of this matrix for an ***olivine*** crystal (orthorhombic) are specified as follows:-

$$\left.\begin{matrix} C_{11} = 3.00 & C_{22} = 1.80 & C_{33} = 2.10 \\ C_{23} = 0.69 & C_{31} = 0.61 & C_{12} = 0.60 \\ C_{44} = 0.58 & C_{55} = 0.71 & C_{66} = 0.70 \end{matrix}\right\} \times 10^{11}\ \text{Pa}$$

referred to the orthorhombic crystal axes x_1, x_2, x_3. ***Write out the matrix C_{rs} in full.***

c) An olivine single crystal is subjected to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 0 \end{bmatrix} \times 10^{-4} \quad \text{(referred to } x_1, x_2, x_3\text{)}$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

d) Calculate also the mechanical work done on the olivine crystal when subjected to the above strain.

e) Write down the first and second laws of thermodynamics and use these to show how the Helmholtz free energy of the crystal is changed as a result of the work done upon it, assuming that the strain is imposed at constant temperature (heat exchange with surroundings is easy).

f) Explain why the Gibbs free energy is not suitable for describing the thermodynamic state of an elastically deformed solid.

Question 2

a) Explain the concept of the Boltzmann distribution law and why this is important in determining the rate of processes such as vacancy migration.

b) Write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T (K) and hydrostatic pressure P. Define all symbols appearing !!

c) Explain the statistical meaning of your answer to part (b), with reference to the Boltzmann distribution law.

d) Show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n superimposed on the hydrostatic component P, and hence explain the theoretical basis (driving force) for solid state diffusion creep.

equal contribution to the total strain rate is provided by both mechanisms). Use this idea to obtain a relationship between recrystallized grain size and flow stress for a pure material deforming by high temperature, climb-controlled dislocation creep plus Nabarro-Herring Creep.

e) Does your result obtained in part (d) resemble experimental observations on recrystallized grain size stress relationships or not, and do you think the model is a good alternative to the Avrami aproach?

Question 7

a) Measured values of the tensile fracture strength (T_o) of brittle materials are usually much lower than the theoretical ("bond strength) value σ_T. Why is this?

b) Consider a flat "elliptical" crack (length 2a) within an infinite plate of elastic material (Young's Modulus E) and suppose that this plate (which is of unit thickness) is subjected to a remote uniaxial tensile stress (σ) oriented normal to the crack surface. The applied stress will give rise to a stored elastic energy U_ε within the plate. Given that the rate of change of U_ε with respect to crack length can be written

$$\frac{d\,U_\varepsilon}{da} = \frac{-2\pi a \sigma^2}{E}$$

derive the Griffith failure criterion for uniaxial tensile loading. **State any assumptions made.**

c) Go on to explain what is meant by the terms "stress intensity factor" and "critical stress intensity factor".

d) Explain what is meant by subcritical crack growth.

e) A sample of low porosity but still permeable quartzite is loaded uniaxially, at room temperature and under dry conditions, to 80% of its brittle compressive failure strength. An initial instantaneous shortening of 0.3% occurs when the load is applied. Over a period of 3 weeks thereafter, no time dependent deformation occurs – the sample length is unchanged. The pores of the rock are then flooded with water at atmospheric pressure and acoustic emission signals are detected, which continue with time. After 1 further week, the sample fails in a brittle manner with no increase in load. Draw a strain-time diagram illustrating how you think the sample behaved as a function of time, and provide an explanation for its behaviour.