Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

14-04-2004 14.00-17.00 hours Kleine Collegezaal

Note:

- The duration of this exam is 3 hours
- The examination paper has two parts:
  -Part A (Deformation, Spiers) has 6 questions; <u>choose any 3</u>
  -Part B (Transport, Peach) has 4 questions; <u>choose any 2</u>
- Hence, answer 5 questions in total.
- Allow about 35 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you identify all mathematical symbols used in answering the questions (marks will be deducted for unidentified symbols).
- Use SI units unless otherwise specified.

Good luck all !!!

<u>PART A</u> (Choose any 3 questions)

<u>Question A-1</u>

a) Using <u>MATRIX NOTATION</u>, write down the stress-strain relations for an anisotropic elastic material.

b) Taking into account the symmetry of the stiffness matrix (C_{rs}), the non-zero components of this matrix for a spinel single crystal (cubic) are specified as follows:-

$C_{11} = C_{22} = C_{33} = 24 \times 10^{10}$ Pa

$C_{44} = C_{55} = C_{66} = 6 \times 10^{10}$ Pa

$C_{12} = C_{13} = C_{23} = 4 \times 10^{10}$ Pa

referred to the cubic crystal axes x_1, x_2, x_3.

Write out the matrix C_{rs} in full, and state the physical significance of the **last** vertical column of this matrix.

c) A spinel single crystal is subjected to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix} \times 10^{-4} \quad \text{(referred to } x_1, x_2, x_3\text{)}$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

d) Calculate also the mechanical work done on the spinel crystal when subjected to the above strain.

e) Write down the first and second laws of thermodynamics in equation form and identify all terms.

f) Use the first and second laws to show how the <u>*Helmholtz free energy ΔF*</u> and <u>*internal energy ΔU*</u> of the crystal change as a result of the work done upon it, assuming that the strain is imposed at constant temperature (heat exchange with surroundings is easy).

Question A-2

a) Explain what vacancies are, and write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T(K) and hydrostatic pressure P. Define all symbols appearing !!

b) Explain the statistical meaning of your answer to part (a), with reference to the Boltzmann distribution law.

c) Show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n superimposed on the hydrostatic component P, and hence explain the theoretical basis (driving force) for solid state diffusion creep.

d) Derive a simple creep model for deformation of a polycrystalline material (grain size d) by Nabarro-Herring creep (lattice diffusion controlled creep). Consider a state of pure shear stress of magnitude σ producing a pure shear strain rate $\dot{\varepsilon}$, and suppose that the grains are square in 2-D.

e) In what regions of the Earth do you expect this type of creep mechanism to be most likely to operate?

Question A-3:

a) Explain what is meant by the terms "superplasticity" (i.e. structural superplasticity) and "necking instability", emphasizing the rheological cause of these phenomena.

b) What are the principal rheological and microstructural characteristics exhibited by crystalline materials that show superplastic deformation behaviour?

c) Explain in detail what microphysical mechanisms give rise to superplastic flow in metals, ceramics and rocks.

d) Mylonitic shear zone rocks often show evidence for the operation of the processes responsible for superplastic flow. Explain how such behaviour can become localized in shear zones and outline the implications for bulk crustal or lithospheric strength.

Question A-4:

a) Gibbs' equation for the condition of chemical equilibrium at (any point on) the interface between a solid and its solution phase is written

$$\mu_s = \mu_n^{eq} \equiv f^s + p\Omega^s + 2\gamma_{SL}\Omega^s / r$$

Explain the meaning of the symbols and terms in this equation.

b) Consider a granular pack of loose mineral grains (diameter d), in which the pores are filled with a saturated solution or partial melt phase. Assume that the the system is characterised by an equilibrium dihedral angle of 45 degrees. If the solid skeleton is instantaneously subjected to an externally applied isotropic stress P and the liquid to an equal hydrostatic pressure $p = P$, describe how the pore structure and permeability of the system will evolve. Illustrate your answer with suitable diagrams and equations (where possible).

c) After the above system reaches equilibrium, the isotropic pressure P applied to the solid skeleton is increased, while the fluid pressure p remains constant. If P is increased sufficiently to remove the system from equilibrium, describe what will happen. Assume that the solid grains can deform elastically but not plastically, as you would expect for say quartz under upper crustal conditions. Illustrate your answer with suitable diagrams and equations (where possible).

d) Explain the importance of such processes in geological systems.

Question A-5

a) Explain, in terms of dislocation dynamics, what is meant by the terms "work hardening" and "recovery".

b) Go on to describe how these processes can lead to steady-state creep in crystalline materials.

c) Develop a rate model for deformation by cross-slip controlled dislocation creep, where the obstacles to dislocation glide are attractive junctions (dipoles) between screw dislocations of opposite sign. Assume that the cross-slip event is a simple, stress-assisted, thermally activated process (similar to that involved in obstacle controlled glide).

Question A-6

a) Measured values of the tensile fracture strength (T_o) of crystalline materials are usually much lower than the theoretical tensile fracture strength σ_T. Why is this?

b) Consider a flat "elliptical" crack (length 2a) within an infinite plate of elastic material (Young's Modulus E) and suppose that this plate (which is of unit thickness) is subjected to a remote uniaxial tensile stress (σ) oriented normal to the crack surface. The applied stress will give rise to a stored elastic energy U_e within the plate. Given that the rate of change of U_e with respect to crack length can be written

$$\frac{dU_e}{da} = \frac{-2\pi a \sigma^2}{E}$$

derive the Griffith failure criterion for uniaxial tensile loading, **stating any assumptions made.**

c) Go on to explain what is meant by the terms "stress intensity factor" and "critical stress intensity factor" for a Mode I crack. Illustrate your answer with suitable equations and diagrams.

d) There are two types of crack extension laws in fracture mechanics, describing time independent and time dependent failure. Explain what these crack propagation laws are, illustrating your answer with specific examples of the associated microscale mechanisms.

PART B (Choose any 2 questions)

Question B-1

a) Darcy's empirical law, for fluid flow through porous media, works well in most, near-surface, geological environments. What are the main limitations to the application of this law, when it is used more generally over a wider range of geological environments, such as hydrothermal and metamorphic systems at elevated temperature and pressure? List these limitations with respect to the pore-fluid properties, the rock properties and their interactions.

b) In addition to isotopic analyses that support high fluid fluxes, microscopic examination of a sample of quartzofeldspathic-schist clearly shows pervasive features of fluid assisted mass transport that also suggest it may have had a significant permeability to the fluid, during its deformation. What prevents us from measuring this permeability in the laboratory, assuming we have a permeameter that could cover the range?

c) A rock has permeability of 1 darcy ($\sim 10^{-12}$ m^2) when tested using water with a viscosity of 1 mPa s. What is the permeability of the rock to oil that has a density 0.8 of the density of water and a dynamic viscosity twice that of water, at the same temperature?

$$\frac{Q}{A} = \frac{\kappa}{\eta} \frac{\Delta P}{L}$$

d) A long-term hydrostatic compaction test at 50 MPa, was carried out on a porous, fine-grained, rock salt aggregate in a pressure vessel. During compaction, the permeability was measured intermittently by a simple "Darcyian" flow-through method using saturated brine, as pore fluid, under a constant differential fluid pressure of 1 MPa, arranged as +0.5 MPa upstream and –0.5 MPa downstream of the aggregate, on a mean background fluid pressure of 10 MPa. The brine flow-through was measured by the displacement of the upstream and downstream servo-controlled pumps, that were used to apply the fluid pressure gradient. It was noted that the compaction rate increased during the periods of permeability measurement, although the mean pore fluid pressure and therefore the mean effective stress, was maintained constant at all times. Why did the permeability determination affect the compaction-rate? What method of permeability measurement would improve the independence of measurement from the compaction process?

Question B-2

a) Explain why natural thrusts often occur, at considerable depth on almost horizontal fault planes, when the coefficient of friction for most crustal rocks (as seen in Byerlee's law) is ~0.8, which carries the implication that unnaturally high and prohibitive horizontal shear stresses are required for the faults to form by shear failure?

b) Explain the factors controlling the slopes and morphology of accretionary wedges.

Question B-3

a) Subterranean pore-fluid pressure changes and earthquakes are intimately linked. Explain this link with the aid of a Mohr diagram for shear failure.

b) What are the concepts of seismic pumping and fault valve behaviour?

c) How do these processes produce mineral veins? What are the geometrical relations between the faults and veins? Using a diagram indicate where the sources and sinks for the minerals are, relative to the fault?

Question B-4

Fully discuss how networks of fractures, produced by progressive hydro-fracture, may have fractal geometries and scaling behaviour controlled by percolation (connectivity) thresholds and growth laws. In your discussion, explain the concept of fractal dimension in terms of size and scale, and then explain how the fractal dimension of quartz-filled fracture systems may possibly relate to their connectivity?