Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

Part II (Spiers) 18-04-2008 09.00 -12.00 hours C.008/010

Note:

- The duration of this exam is 3 hours.
- Answer any 4 of the 7 questions given.
- All questions count with equal weight to the final grade.
- Allow about 45 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you identify all mathematical symbols used in answering the questions (marks will be deducted for unidentified symbols).
- Use SI units unless otherwise specified.

Good luck all !!!

Question 1

a) Taking into account the symmetry of the elastic stiffness matrix (C_{rs}), the non-zero components of this matrix for an ***olivine*** crystal (orthorhombic) are specified as follows:-

$$\left.\begin{array}{lll} C_{11} = 3.0 & C_{22} = 1.8 & C_{33} = 2.1 \\ C_{23} = 0.69 & C_{31} = 0.61 & C_{12} = 0.60 \\ C_{44} = 0.58 & C_{55} = 0.71 & C_{66} = 0.70 \end{array}\right\} \times 10^{11}\ \text{Pa}$$

referred to the orthorhombic crystal axes x_1, x_2, x_3. Write out the matrix C_{rs} in full.

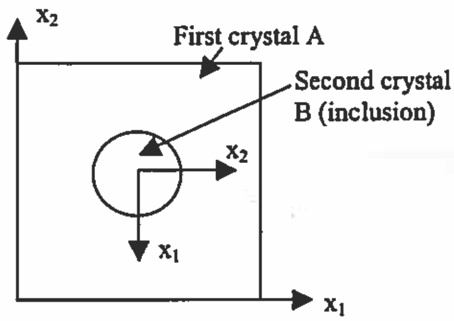
b) An olivine single crystal is deformed at constant temperature to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \times 10^{-4} \quad \text{(referred to its crystal axes } x_1, x_2, x_3\text{)}$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

c) Calculate the change in the Helmholtz free energy of the crystal.

d) Suppose that the crystal (A) contains a second olivine crystal (B) as an inclusion, and that the crystal axes of this included crystal are orientated as shown in the figure on the right. If the strain applied to the first (outer) crystal is uniform throughout both crystals (homogeneous strain field), what is
-the state of strain in B, specified with respect to its own crystal axes?
-the state of stress in B, specified with respect to its own axes?
-the Helmholtz free energy of B?



e) If grain boundary migration is possible, in the above configuration, will B grow or shrink?

Question 2

a) Explain what vacancies are, and write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T (K) and hydrostatic pressure P. Define all symbols appearing !!

b) Explain the statistical meaning of your answer to part (a), with reference to the Boltzmann distribution law.

c) Show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n superimposed on the hydrostatic component P, and hence explain the theoretical basis (driving force) for solid state diffusion creep.

d) List the steps you would go through to derive a simple creep model for deformation of a polycrystalline material (grain size d) by Coble creep (grain boundary diffusion controlled creep). There is no need to derive the model in detail.

e) Write down the theoretical rate equation for Coble creep and show how the viscosity of rock deforming by this mechanism is expected to depend on temperature and pressure.

Question 3

a) Explain what is meant by the terms "necking instability" and "superplasticity" (structural), indicating the rheological characteristics of materials showing such behaviour.

b) List the mechanisms of steady state flow that can give rise to necking instability.

c) List the principal microstructural characteristics exhibited by crystalline materials that show "superplastic flow", and explain which deformation mechanisms can lead to such behaviour.

d) Mylonitic shear zone rocks often show evidence for the operation of the processes responsible for superplastic flow. Explain three ways in which such behaviour can become localized in shear zones and outline the implications for the strength of major shear zones compared to surrounding host rock.

e) List the problems faced by materials scientists seeking to promote superplastic behaviour in metallic alloys and ceramics, and explain how these problems can be overcome!

Question 4

a) Write down Gibbs' equation giving the condition of chemical equilibrium at any point on a curved interface between a solid and its solution phase. Go on to explain how the local equilibrium chemical potential of dissolved solid can be influenced by deformation of the solid phase.

b) Consider a granular pack of (isotropic) mineral grains (diameter d) in which the pores are filled with a saturated solution or partial melt phase. The solid skeleton is simultantaneously subjected to an externally applied hydrostatic stress P and the liquid to an equal hydrostatic pressure $p = P$. Describe how the system will respond, illustrating your answer with realistic diagrams. Assume that the the system is characterised by an equilibrium dihedral angle of 75 degrees.

c) The above system is allowed to reach equilibrium. What will its permeability be in the equilibrium state?

d) The externally applied hydrostatic stress P is now increased substantially, so that $P >> p$. What will tend to happen now?

e) What is the is the importance of the type of process you have described ?

Question 5

a) Explain, in terms of dislocation dynamics, what is meant by the terms "work hardening" and "recovery".

b) Describe in detail how these processes can lead to steady-state creep in rocks at elevated temperatures.

c) Develop a rate model for deformation by high-temperature, cross-slip-controlled dislocation creep where the obstacles to dislocation glide are attractive/repulsive junctions between screw dislocations of opposite sign (assume a linear dependence of the activation barrier upon stress, as for discrete obstacle controlled glide).

d) Dislocation creep in calcite at high temperatures shows an apparent power law n-value of 7 or 8. Indicate whether this suggests climb or cross-slip controlled creep, and explain why.

Question 6

a) Explain what is meant by the term "grain boundary migration" and indicate what thermodynamic forces can drive the process.

b) Derive an expression for the velocity of migration (V) of a grain boundary separating two grains in a pure crystalline solid, assuming a free energy difference between the grains of Δf.

c) Explain how and why this expression would be modified if the material contained a solution phase in grain boundaries.

d) By comparing the likely activation energies for grain boundary migration in dry and wet materials (in a V vs. $1/T$ diagram), indicate the conditions under which you expect fluid assisted grain boundary migration to dominate over solid state migration in pure solids.

e) How would you recognise if a natural rock has recrystallised by fluid assisted grain boundary migration as opposed to solid state migration?

Question 7

a) Measured values of the tensile fracture strength (T_o) of rocks and ceramics are usually much lower than the theoretical tensile fracture strength σ_T. Why is this?

b) Consider a flat "elliptical" crack (length $2a$) within an infinite plate of elastic material (Young's Modulus E) and suppose that this plate (which is of unit thickness) is subjected to a remote uniaxial tensile stress (σ) oriented normal to the crack surface. The applied stress will give rise to a stored elastic energy U_e within the plate. Given that the rate of change of U_e with respect to crack length can be written

$$\frac{dU_e}{da} = \frac{-2\pi a \sigma^2}{E}$$

derive the Griffith failure criterion for uniaxial tensile loading, **stating any assumptions made.**

c) Go on to explain what is meant by the terms "stress intensity factor" and "critical stress intensity factor" for a Mode I crack.

d) List the principal microscale mechanisms of catastropic fracture in polycrystalline materials, explaining how the flaws causing failure are generated in each case. Illustrate your answer with simple diagrams.

e) A wet rock sample is loaded at room temperature and at a stress which produces around 0.5% instantaneous shortening. The applied stress is too low to cause immediate fracture, but acoustic emissions are recorded immediately upon application of the load. Acoustic emissions subsequently continue with time and the sample deforms by slow, on-going creep. It finally fractures after 2 weeks. Offer an explanation for the behaviour observed, illustrating your answer with a strain-time plot and making use of any simple formulas that are relevant.