Principles of Groundwater Flow

November 6, 2009

13:00-16:00

Success!

1. In the vicinity of Amsterdam, the biggest and most polluted dump site for high-level chemical waste is located in the Volgermeer Polder. In the early sixties of the last century, several well-known Dutch companies were given permission [1] by the local Amsterdam authorities to dump their toxic waste in the (former) peat excavation area. This went on for many years and the high-level chemical waste was covered by soil and domestic waste. In March 1980, an new employee working at the site found a barrel filled with extremely toxic waste. When he informed the press, he immediately was fired... However, his action was the starting point for the awareness of the huge environmental problem in the Volgermeer Polder. The site is too large for excavation and remediation, so hydrogeological confinement seems to be the best option. As long as the contaminants do not migrate in the soil-groundwater system, it's OK.

   The basic idea is to cover the chemical waste with a impermeable foil such that no rainwater can enter the dump site. In addition, the foil is covered with a new peat layer, which is assumed to take over the water-retaining function of the foil when the life-time of the foil is exceeded and the foils starts leaking (rain)water.

   Let's consider a (very) simplified version of the situation in the Volgermeer Polder. We assume that the soil is homogeneous (peat). The aquifer is phreatic (unconfined) and we assume a confining layer at a certain depth. We consider a vertical cross section. The dump site is covered form $x = 0$ to $x = x_0$ by an impermeable foil. From $x = x_0$ to

[1] Although it was NOT allowed by law!

$x = L$ rain replenishes (recharges) the aquifer with a constant rate N [m/s]. At $x = L$ a ditch (small draining canal) is located. See Figure 1. Hydraulic conductivity of the soil is k [m/s]. The governing stationary unconfined differential equation is

$$k\frac{d}{dx}\left(h\frac{dh}{dx}\right) = -N \tag{1}$$

where k is hydraulic conductivity [m/s], N the recharge flux (rain) [m/s] and $h = h(x)$ is the height of the groundwater table [m]. Lets assume that there is **no flow** under the area covered with the foil. This implies a no-flow boundary such that

$$\text{at} \quad x = x_0 \quad \text{we have} \quad \frac{dh}{dx} = 0$$

The hydraulic head (i.e. water level) in the ditch is kept constant: $h(L) = h_L$.

(a) Show by derivation that the hydraulic head distribution $h = h(x)$ in the region from $x = x_0$ to $x = L$ is given by

$$h^2(x) = h_L^2 - \frac{N}{k}(x^2 - L^2) + \frac{2N}{k}x_0(x - L) \tag{2}$$

(b) Derive an expression for $Q' = Q'(x)$, i.e. the volumetric flow rate per unit width of the aquifer

(c) Show that the $Q'(L)$, i.e. the amount of water that is drained by the ditch at $x = L$ is given by

$$Q'(L) = (L - x_0)N$$

(d) Explain in words why the simple expression found under (c) must be correct from the hydrological point of view

(e) Use expression (2) to write down an expression for the height of the water table at $x = x_0$. Using this result we are going to analyze two different limiting situations:

i. $x_0 = 0$, implying NO FOIL cover of the dump site and all recharge enters the aquifer

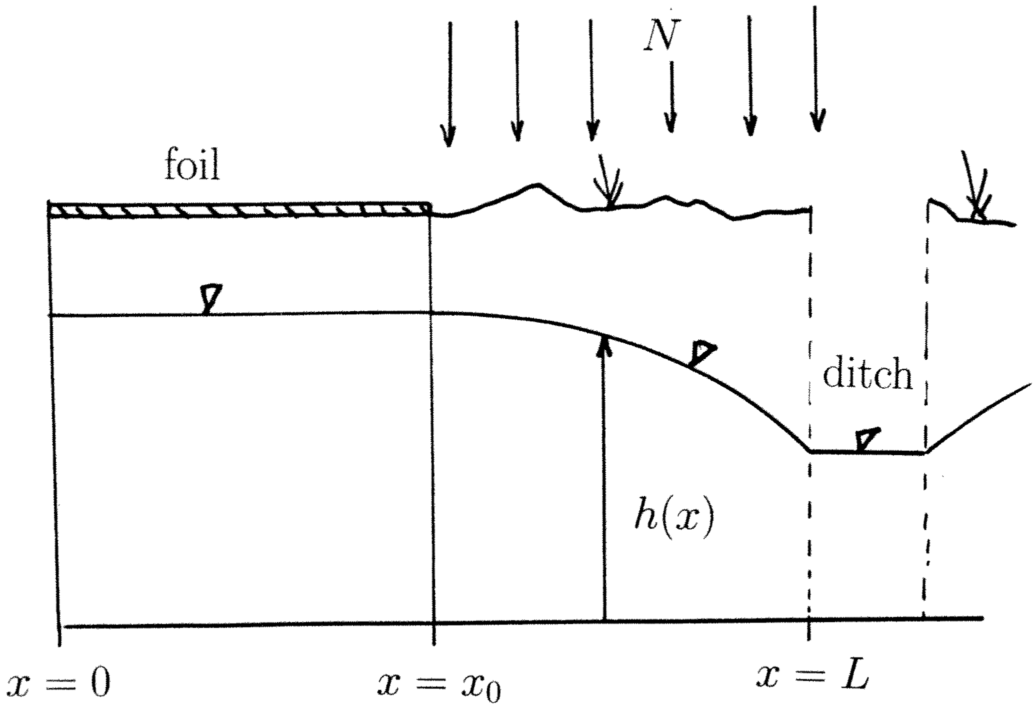


Figure 1 Partially covered phreatic aquifer

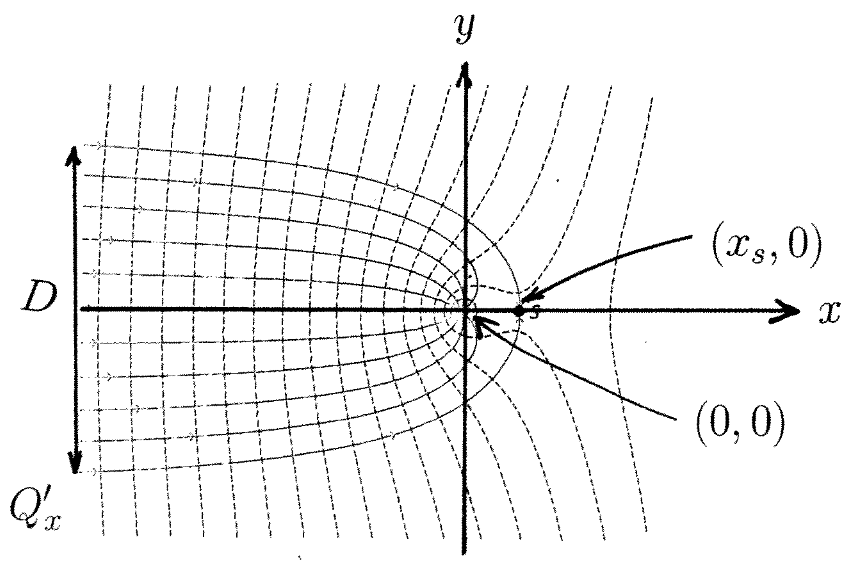


Figure 2 Unconfined uniform flow with pumping well

(f) The position of the so-called stagnation point is given by $(x, y) = (x_s, 0)$. At this point the hydraulic gradient in the x-direction is zero: $dh/dx(x_s, 0) = 0$. Show that

$$x_s = \frac{Q_w}{2\pi Q'_x} = \frac{D}{2\pi}$$

Hint: Let $y = 0$ in expression (4) and determine the derivative dh/dx, etc, etc ...

3. Indicate whether the following statements are correct or not, and motivate your answer

   (a) For small density differences, the fluid mass balance equation can be replaced by the continuity equation. This is the so-called Boussinesq approximation

   (b) A homogeneous aquifer is never isotropic

   (c) Flow lines and hydraulic head contours are always perpendicular

   (d) In confined aquifers, changes in σ_{ve}, i.e. the vertical effective stress, are always small and therefore disregarded

   (e) A no-flow boundary in porous media flow is not a flow line

   (f) Quicksand or liquefaction occurs when the vertical effective stress exceeds the fluid pressure in the pores

   (g) In general the compressibility of the sediment is higher than the compressibility of the fluid

4. We consider a confined aquifer. Previous pumping tests showed that the transmissivity and storativity of the aquifer are $T = 1500$ m^2/day and $S = 0.0002$ respectively. Two observation wells are installed in the aforementioned aquifer. Observation well A is located at $r_A = 250$ m from a pumping well, while observation well B is located at $r_B = 500$m from the pumping well. At $t = 0$ the pumping well starts to operate at constant volumetric flow rate $Q = 500$ m^3/day. Hint: use Theis solution and corresponding well-function graph.

   (a) Determine the drawdown in observation well **A** after pumping $t = 300$ minutes

ii. $x_0 = L$, implying that the whole region is covered by foil and no discharge of rain in to the ditch

Determine for BOTH situations the height of the groundwater table at $x = 0$, i.e. $h(0)$.

2. Consider uniform constant flow in the positive x-direction in an unconfined aquifer. The aquifer has a constant hydraulic conductivity k [m/s]. The general expression for the hydraulic head is given by

$$h^2(x, y) = Ax + C_1 \tag{3}$$

where A and C_1 are constants

(a) The volumetric flow rate in the x-direction per unit width in the y-direction is Q'_x, which is assumed to be constant. Use the definition of Q'_x to show that

$$A = -\frac{2Q'_x}{k}$$

(b) Next we assume that a stationary pumping well is present in the origin of our (x,y)-coordinate system. The well has constant pumping rate Q_w and well radius is r_w. Show that the hydraulic head in an arbitrary point (x, y) satisfies the general expression:

$$h^2(x, y) = -\frac{2Q'_x}{k}x + \frac{Q_w}{\pi k}\ln(r) + C_2$$

(c) The hydraulic head in the well is $h(r_w) = h_w$. Show that the hydraulic head in an arbitrary point (x, y) is given by

$$h^2(x, y) = h_w^2 - \frac{2Q'_x}{k}(x - r_w) + \frac{Q_w}{\pi k}\ln\left(\frac{\sqrt{x^2 + y^2}}{r_w}\right) \tag{4}$$

(d) Explain the hydrogeological significance of a well capture zone

(e) Let's assume that the width of the well capture zone far away from the pumping well is given by D in [m]. See Figure 2. Explain why the following expression must be correct:

$$Q'_x D = Q$$

Formula sheet

Transient flow

$$h = h_0(x, y) - \frac{Q}{4\pi T} W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-m}}{m}\, dm$$

Approximation of $W(u)$ (truncated series)

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{4} + \frac{u^3}{18} - \frac{u^4}{96}\ldots$$

$$\gamma = 0.5772157\ldots \text{ Euler's constant}$$

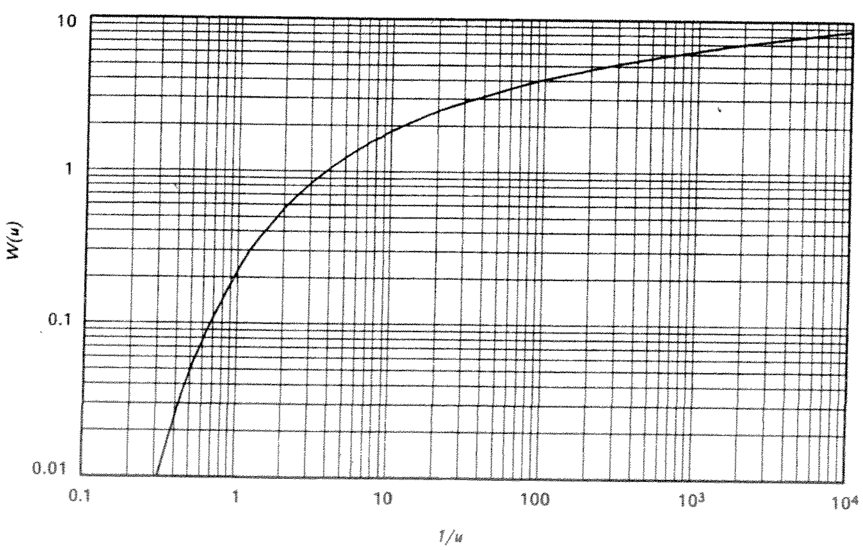
$$u = \frac{r^2 S}{4T(t - t_0)}$$

Radial flow in a confined aquifer

$$h(r) = \frac{Q}{2\pi KD} \ln r + C_1$$

Radial flow in an unconfined aquifer

$$h^2(r) = \frac{Q}{\pi K} \ln r + C_2$$



(b) Determine the drawdown in observation well **B** after pumping $t = 300$ minutes

(c) What **new** pumping rate Q would be required to establish a drawdown of 0.5 meter in observation well B after 300 minutes of pumping?

(d) If both T and S would attain **half** their original values, how would this alter your answers found under (a) and (b)? You are allowed to give a quantitative answer (simply compute it) or qualitative (explain in words)

(e) If $B = 50$ m, porosity is $n = 0.35$, fluid density is $\rho = 1000$ kg/m^2 and dynamic viscosity is $\mu = 0.001$ kg/(ms), compute the intrinsic permeability of the sediment.