1 + 2

Midterm Exam

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October 4, 2012

1 Theory

In continuum mechanics, stress is a measure of the internal forces acting within a deformable body. It is represented mathematically by a tensor that can be represented in 2D and 3D by 2×2 or 3×3 matrices respectively:

$$\boldsymbol{\sigma}_{2D} = \left(\begin{array}{cc} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{array} \right) \qquad \boldsymbol{\sigma}_{3D} = \left(\begin{array}{ccc} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{array} \right)$$

One can define the trace of the stress tensor which is given by

$$Tr(\boldsymbol{\sigma}_{2D}) = \sigma_{11} + \sigma_{22}$$

$$Tr(\boldsymbol{\sigma}_{3D}) = \sigma_{11} + \sigma_{22} + \sigma_{33}$$

Also, the determinant of the tensor is given by

$$\begin{array}{rcl} D(\boldsymbol{\sigma}_{2D}) & = & \sigma_{11}\sigma_{22} - \sigma_{21}\sigma_{12} \\ D(\boldsymbol{\sigma}_{3D}) & = & \sigma_{11} * (\sigma_{22}\sigma_{33} - \sigma_{32}\sigma_{23}) \\ & - & \sigma_{21} * (\sigma_{12}\sigma_{33} - \sigma_{32}\sigma_{13}) \\ & + & \sigma_{31} * (\sigma_{12}\sigma_{23} - \sigma_{22}\sigma_{13}) \end{array}$$

Applying a stress $\boldsymbol{\sigma}$ on a solid, the force exerted on its surface defined by a normal vector $\boldsymbol{n}_{2D} = (n_1, n_2)$ or $\boldsymbol{n}_{3D} = (n_1, n_2, n_3)$ is given by

$$\boldsymbol{f} = \boldsymbol{\sigma} \cdot \boldsymbol{n}$$

or,

$$\boldsymbol{f}_{2D} = \left(\begin{array}{c} f_1 \\ f_2 \end{array} \right) = \left(\begin{array}{c} \sigma_{11}n_1 + \sigma_{12}n_2 \\ \sigma_{21}n_1 + \sigma_{22}n_2 \end{array} \right) \qquad \boldsymbol{f}_{2D} = \left(\begin{array}{c} f_1 \\ f_2 \\ f_3 \end{array} \right) = \left(\begin{array}{c} \sigma_{11}n_1 + \sigma_{12}n_2 + \sigma_{13}n_3 \\ \sigma_{21}n_1 + \sigma_{22}n_2 + \sigma_{23}n_3 \\ \sigma_{31}n_1 + \sigma_{32}n_2 + \sigma_{33}n_3 \end{array} \right)$$

The stress tensor is said to be symmetric when for all couples (i, j)

$$\sigma_{ij} = \sigma_{ji}$$

One can obtain the symmetric part of a tensor by computing: $\boldsymbol{\sigma}^s = \frac{1}{2}(\boldsymbol{\sigma} + \boldsymbol{\sigma}^T)$ where $\boldsymbol{\sigma}^T$ is the transpose of the stress tensor given by $\sigma_{ij}^T = \sigma_{ji}$.

2 Assigment

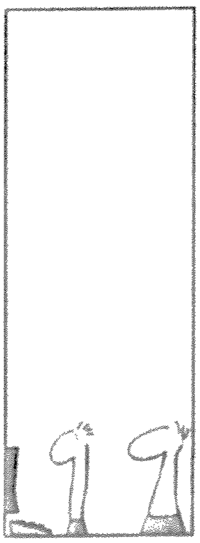
Write a fortran program according to the following specifications:

1. (1pt) declare the following arrays
   `force` : 1D allocatable array
   `stress` : 2D allocatable array
   `stressT` : 2D allocatable array
   `stress_sym` : 2D allocatable array
2. (1pt) read from the standard input the number of dimensions `ndim`
3. (1pt) allocate the arrays to their correct sizes

4. (1pt) fill the stress tensor with random numbers
5. (2pt) write a simple subroutine which takes the two arrays `stress` and `stressT` as arguments and returns the transpose of `stress` in `stressT`
6. (1pt) compute the symmetric part of the stress tensor and store it in `stress_sym`
7. (1pt) compute the trace `Tr` and the determinant `Det` of the stress tensor
8. (1pt) $\boldsymbol{n}_{2D} = (-1, -1)$ in 2D or $\boldsymbol{n}_{3D} = (-\cos(\theta), \sin(\theta), \tan(\theta))$ with $\theta = 0.5$ in 3D. Compute the force exerted on this surface.
9. (1pt) write in file *measurement.dat* the number of dimensions, the trace, the determinant, and the computed force.

<u>Recommendations</u>:

- Keep in mind that the code has to work for `ndim`=2 or 3, and that it should not work for `nd`=-1, or `nd`=4. (`If-then-else` constructions might be of use here throughout the program)
- Every single used variable has to be defined.
- Comment your code appropriately.
- Points will be deduced for unclear/unreadable statements.
- Every single used variable has to be defined (I insist).



Good luck