Principles of Groundwater Flow

November 13, 2007
17:00-20:00

1. We consider pumping test analysis in a confined aquifer with constant thickness D m. The confining clay layers can be considered as non-leaky. The test is conducted at a constant volumetric flow rate Q m^3/day. Two observation wells A and B are constructed: i.e. respectively at $r_A = 25$ m and at $r_B = 50$ m distance from the production well. The series expansion of Theis solution can be writte as

$$h_0 - h = \frac{Q}{4\pi T}\left(-\gamma - \ln u + u - \frac{u^2}{4} + \frac{u^3}{18} - \frac{u^4}{96}\cdots\right)$$

where $\gamma \approx 0.57721$ (= Euler's constant).

   (a) This series expansion can be truncated under a certain assumption, yielding

$$h_0 - h \approx \frac{Q}{4\pi T}\left(-\gamma - \ln u\right)$$

   What is this assumption and explain and motivate the physical/hydrological consequence(s) of this assumption/approximation

   (b) After substitution of the definition of the variable u, and after some algebra we obtain

$$h_0 - h \approx \frac{\ln(10)\,Q}{4\pi T}\left[\log(t - t_0) + \log\left(\frac{4T}{e^{\gamma} r^2 S}\right)\right]$$

   Let us assume that the draw down values in both observation wells, i.e. $(h_0 - h)_A$ and $(h_0 - h)_B$, are measured at the same time instant $(t - t_0)$, where t is the pumping time since the pumping started at t_0. Show by derivation that in case of two observation wells located at r_A and r_B meter distance respectively from the pumping well that the following equality holds:

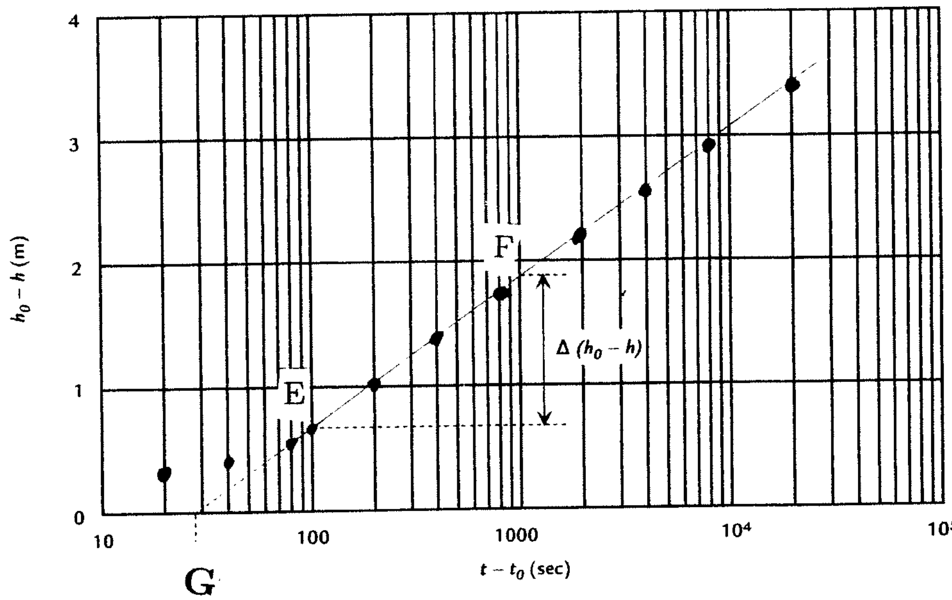
$$(h_0 - h)_A - (h_0 - h)_B \approx \frac{Q}{2\pi T}\ln\frac{r_B}{r_A}$$

   Hint: Note that $\log\left(\frac{4T}{e^{\gamma} r^2 S}\right) = \log\left(\frac{4T}{e^{\gamma} S}\right) + \log\left(\frac{1}{r^2}\right)$ **and** that $\ln(10)\log(\frac{r_b}{r_a}) = \ln(\frac{r_b}{r_a})$.

(c) Explain this result in hydrogeological terms.

(d) If $Q = 2\ \mathrm{m}^3/\mathrm{min}$ and the observed hydraulic head difference at a certain time instant $(h_0 - h)_A - (h_0 - h)_B \approx 0.67$ m, determine the transmissivity T of the aquifer in m^2/s using the approximate expression under (b).

(e) Next we ONLY consider the time-dependent drawdown in well B to determine the storativity. The drawdown data are plotted on a semi-log scale and shown in Figure 1.



Semilog plot of $h_0 - h$ vs. $t - t_0$.

Figure 1. Drawdown at the location of observation well B (semi-log scale)

For large times the drawdown in the observation well is linear at this semi-log scale, satisfying the expression

$$h_0 - h = M[\log(t - t_0) + N]$$

Determine M and N in terms of the parameters Q, T, r, etc.

(f) Explain why the data in Figure 1. for short observation times show a nonlinear tendency.

(g) In the graph we have picked two points E and F, such that $\log(t-t_0)_F - \log(t-t_0)_E = 1$. Show that

$$T = \frac{\ln(10)Q}{4\pi\Delta(h_0 - h)},$$

where $\Delta(h_0 - h) = (h_0 - h)_F - (h_0 - h)_E$.

(h) Compute T using this expression and compare the result with your findings under (d).

(i) The intersection of the horizontal axis and the extrapolation of the straight line through the data points for large times can be determined from Figure 1. For convenience we refer to this point as G (in seconds). Show that storativity S can be computed using from the expression

$$S = \frac{4TG}{e^{\gamma} r_B^2}$$

(j) Determine S using the expression under (i).

(k) Theis solution is valid under a large variety of assumptions. List at least four of these assumptions.

2. A pumping well is located 100 m from the shore of a river that is roughly straight near the well. The well bore diameter is $d = 0.8$ m. Let's assume that the filter is in direct contact with the aquifer sediment. (This is usually not the case of course). Both the well screen and the river are in direct contact with the same confined aquifer, which has an estimated transmissivity of $T = 150$ m^3/day. The effective porosity is $n = 0.4$ and the thickness of the aquifer $D = 10m$.

   (a) Use the method of images (superposition) to show that the stationary hydraulic head in the vicinity of the well/river is given by

   $$h = \frac{Q}{2\pi T} \ln \frac{r_1}{r_2} + C$$

   (b) What is the hydrogeological meaning of the constant C in this expression?

(c) Let's assume that the origin of the coordinate system (x, y) is located in the middle of the straight line connecting the pumping well and the image well. The hydraulic head distribution along this line is given by:

$$h(x) - h_0 = \frac{Q}{2\pi T} \ln \left[\frac{a + x + r_0}{a - x + r_0} \right],$$

where $r_0 = d/2$ denotes the radius of the filter, and a the distance between the the constant head boundary (river) and the outer radius of the filter. Show by derivation that this expression is correct.

(d) Determine the drawdown at the location of the pumping well, assuming it is pumped for a long time at a volumetric flow rate of $Q = 600$ m^3/day.

(e) Determine the shortest(!) travel time for a tracer particle to travel from the shore of the river towards the pumping well. Hint: the travel time of a tracer particle is

$$t_{travel} = \int_0^a \frac{1}{v(x)}\, dx,$$

where v denotes the effective groundwater velocity.

(f) Next we consider the same situation, but now the more realistic case that a gravel pack is present around the aforementioned well filter. The hydraulic conductivity of the gravel pack is $K_g = 150$ m/day (i.e. $T_g = 1500$ m^2/day !) and the effective porosity is $n_g = 0.4$. The outer diameter of the gravel pack is 1.6 m.

i. Do you expect a big change in the drawdown compared to the value you found under (c)?

ii. Give a 'rough' estimate of the new value of the drawdown, based on hydrogeological intuition.

3. Answer the following questions and **motivate** your answer

(a) Atmospheric pressure fluctuations are in general small but cause nevertheless changes in stresses, pore water pressures and water levels in wells. True or not?

(b) The famous stress relationship of Terzaghi (1925) is given by

$$\sigma_{vt} = P + \sigma_{ve}$$

In confined aquifers, the vertical total stress σ_{vt} is always zero. True or not?

(c) In a layered system consisting of n-layers, the vertical total stress can be computed according to:

$$\sigma_{vt} = g \sum_{i=1}^{n} \rho_i D_i,$$

where ρ_i denotes the 'wet' density of the i-th layer, D_i is the thickness of the i-th layer, and g is the acceleration due to gravity. Correct or not?

(d) If the directors of the famous movie Lawrence of Arabia would have attended a proper groundwater or soil mechanics course, would they have removed the scene from their movie where one of the servants of Lawrence, i.e. Daud, drowns in a pool of quicksand? A bad case of Hollywood physics or not?

Figure 2. The unfortunate servant Daud of Lawrence of Arabia

(e) The correct expression for the specific storage S_s is given by

$$S_s = \rho_w g(n\beta + (1 - n)\alpha),$$

where β and α are respectively the compressibility of the fluid and the solid phase. True or not?

4. Consider the vertical cross section of a **phreatic aquifer** with an horizontal impermeable base. The length is $L = 100$ m, and the hydraulic conductivity $K = 1.0\ 10^{-4}$ m/s. The aquifer is bounded by two canals. The canal at the left-hand canal has constant head h_L and the right-hand canal h_R. The whole area is subject to recharge (i.e. rain) N m/s. The effective porosity is $n = 0.35$ and the vertical width D of the aquifer is 35 m, i.e. the distance from the impermeable base towards ground surface.

   (a) Motivate why the strictly horizontal and stationary flow in this phreatic aquifer satisfies the differential equation:

   $$\frac{d}{dx}\left(Kh\frac{dh}{dx}\right) + N = 0$$

   (b) Solve the equation to determine the hydraulic head distribution $h = h(x)$. Do **not** insert any of the given numbers but give the solution in terms of the symbols!

   (c) If $h_L = 30$ m and $h_R = 25$ m, compute the height of the water table at $x = L/2 = 50$ m.

   (d) Determine the position of the water divide.

   (e) Next the first 50 meters at the left-hand side are covered with impermeable material, such that only half of the recharge can enter in the right-hand side part of the aquifer. How much must we lower the level in the left-hand canal such that **ALL** recharge flows towards the left hand canal?

..... The End .....

Formula sheet

Transient flow

$$h = h_0(x, y) - \frac{Q}{4\pi T} W(u)$$

$$W(u) = \int_u^\infty \frac{e^{-m}}{m} \, dm$$

Approximation of $W(u)$ (truncated series)

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{4} + \frac{u^3}{18} - \frac{u^4}{96} \ldots$$

$\gamma = 0.5772157...$ Euler's constant

$$u = \frac{r^2 S}{4T(t - t_0)}$$

Radial flow in a confined aquifer

$$h(r) = \frac{Q}{2\pi KD} \ln r + C_1$$

Radial flow in an unconfined aquifer

$$h^2(r) = \frac{Q}{\pi K} \ln r + C_2$$