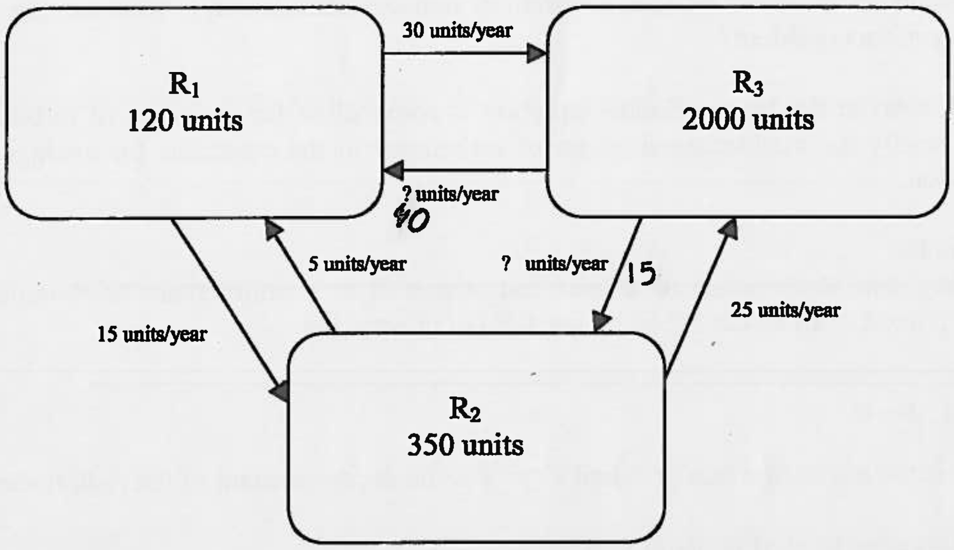
REACTIVE TRANSPORT IN THE HYDROSPHERE

EXAM 12 APRIL 2006

Question 1

Given the following scheme of reservoirs for a biogeochemical cycle in steady state



a) Write the set of linear differential equations describing the system. Use matrix notation! Explain the assumptions which need to be taken for such mathematical representation of the problem.

b) Write the evolution in time of the system (the general solution of the previous set of differential equations) as a function of the theoretical eigenvalues and eigenvectors, using symbols, explaining what the different symbols stand for, and supposing the system was not initially at steady state.

c) What can you say about the sign of the eigenvalues? What does this mean in terms of reaching a steady state (use the functions written in the previous section in your explanation)?

Question 2

a) The stability of a system governed by a set of non-linear differential equations can be studied using the method of perturbation. Explain this method for a generic two-dimensional case. Don't derive the equations, just write them in a generic form, and explain the steps you have to take, and the assumptions and limitations of the method.

b) When dealing with a reactive transport model (RTM), describe what the residual functions represent and where they come from.

c) The solution of the system of equations making the residual functions equal to zero frequently need an iterative method instead of a direct one. Why? Describe succinctly the Newton-Raphson method.

Question 3

a) List the set of conservation equations that are required to solve for the flow field in a marine stratified environment (i.e. a system of non-constant density). What are the set of unknowns in this problem?

b) Which term in the Navier-Stokes equation is responsible for the onset of turbulence? Explain briefly the mathematical origin of turbulence in the equations for average fluid flow motion.

Question 4

The steady state distribution of a solid radiotracer B in a sedimentary environment of constant porosity can be described by the following equation:

$$\omega \cdot \frac{dB}{dx} + k \cdot B = 0$$

where ω is the burial rate (cm.yr^{-1}) and k (yr^{-1}) is the decay constant of the radiotracer

a) Solve for B(x) if $B=B_0$ at x=0.
b) Draw schematically the depth distribution of B(x) for $k=0$ and $k >> \omega$.
c) Which parameter can usually be quantified from such depth distribution of a radiotracer?

Assume that the same solid radiotracer is now subject to mixing (by bioturbation, with bioturbation coefficient D_b), leading to the following mass conservation equation at steady-state:

$$D_b \cdot \frac{d^2B}{dx^2} - \omega \cdot \frac{dB}{dx} - k \cdot B = 0$$

d) What is the assumption made about D_b in the above equation?
e) Find the solution for B(x) if $B=B_0$ at x=0 and dB/dx = 0 as $x \to \infty$
f) Assuming that ω is negligible, plot the depth-distribution of B over the first 50 cm for $k = 100 \cdot D_b$.
g) Write now the mass conservation equation for the transient (i.e. time-dependent) case.
h) What additional information will be required to solve such equation?
i) Write an explicit numerical integration scheme for this last transient case, indicating the type of differences you use for each term. Explain why the scheme is called explicit, and the notation you use well in detail.
j) What is known and what is unknown in the numerical scheme? Explain the evolution of the solution (what is known and what is unknown, and how the equation(s) is (are) solved) of the problem when applying this scheme.