Examination Paper: *Mechanisms of Deformation and Transport in Rocks*

Part II (Niemeijer) 16-04-2021 11h30 -14h00 hours Room: THEATRON

- The duration of this exam is 2.5 hours.
- Answer question **1** plus any **3** of the remaining 6 questions given.
- Allow about 30 minutes per question.
- Answers may be given in English or Dutch.
- Make sure you **identify all mathematical symbols** used in answering the questions
- Use SI units unless otherwise specified.
- If you do not understand the English in any of the questions, raise your hand for help.
- Read the question carefully!
- Note that pages are printed double-sided!

Good luck!

Question 1

Indicate for the following 8 statements whether they apply to dislocation creep or diffusion creep.
Shortly motivate your answer, where possible (a,b, c,d,e,f,g). **Without motivation, no points will be given!**

a) Very stress dependent ($n >> 1$)
b) Formation of a crystallographic preferred orientation (CPO = LPO = lattice preferred orientation)
c) Weertman equation
d) Vacancies
e) Internal grain deformation
f) Grain size sensitive (GSS)
g) Superplasticity
h) Coble creep

Question 2

a) Write down the stress-strain relations for an anisotropic elastic material using **matrix** notation.

b) Taking into account the ***symmetry*** of the stiffness matrix (C_{rs}), the non-zero components of this matrix for a ***Topaz*** crystal (orthorhombic, $Al_2(SiO_4)(F,OH)_2$) are specified as follows:-

$$\left.\begin{array}{lll} C_{11} = 2.8136 & C_{22} = 3.8495 & C_{33} = 2.9452 \\ C_{23} = 0.8815 & C_{31} = 0.8464 & C_{12} = 1.2582 \\ C_{44} = 1.0811 & C_{55} = 1.3298 & C_{66} = 1.3089 \end{array}\right\} \times 10^{11}\ \text{Pa}$$

referred to the orthorhombic crystal axes x_1, x_2, x_3. ***Write out the matrix C_{rs} in full.***

c) Explain the physical meaning of the first vertical column of the matrix C_{rs}.

d) A topaz single crystal is subjected to an elastic strain given by the tensor

$$\varepsilon_{ij} = \begin{bmatrix} 4.5 & 0 & 2 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \end{bmatrix} \times 10^{-4} \text{ (referred to } x_1, x_2, x_3)$$

Use C_{rs} to calculate the resulting state of stress, writing your answer in both matrix and tensor notations.

e) Calculate also the mechanical work done on the anhydrite crystal when subjected to the above strain, making sure you state the units!!

f) Use the first and second laws of thermodynamics to **show** how this mechanical work changes the thermodynamic state of the crystal, if deformation occurs at constant temperature, and explain what would happen to the crystal if it is placed in contact with an unstrained anhydrite crystal with a fluid layer between the two crystals.

Question 3

a) Write down an equation giving the chemical potential of a solid at a point on a fluid-filled grain-to-grain boundary experiencing a normal stress σ_n. Identify all of the symbols used and state their units. Assume the grain boundary contains fluid is in the form of an island-channel structure or as an adsorbed film.

b) Use this equation to derive an equation for the rate of pressure solution creep of a material consisting of densely-packed cubic grains, subject to a pure shear stress of magnitude σ. Assume reaction control (i.e. dissolution/precipitation control).

c) Now write down an equation for the creep rate assuming diffusion control (no need to derive this).

d) By equating the two results you obtain, develop a criterion for determining the conditions under which pressure solution is controlled by reaction versus diffusion.

e) Assuming porosity has little influence on this criterion, use it to determine which mechanism you expect to control pressure solution creep in i) ultrafine fault rocks and ii) a very coarse-grained gas reservoir sandstone

Question 4

e) List at least 2 possible configurations for performing friction experiments, sketch them and list a possible disadvantage of each method.

f) Show why friction is *not* a material property.

g) Go on to explain how shear stress should depend on (effective) normal stress if the contacts obey Hertzian contact mechanics and give a possible explanation why experiments almost always show that shear stress varies linearly with (effective) normal stress in a typical friction experiment.

h) Rate and state friction equations describe the evolution of friction with time (or displacement) and are given by:

$$\mu = \mu_0 + a\ln\left(\frac{V}{V_0}\right) + b\ln\left(\frac{V_0\theta}{d_c}\right)$$

$$\frac{d\theta}{dt} = 1 - \frac{\theta V}{d_c}$$

Derive an expression for determing the value of (a-b) from steady state friction measurements using the slowness evolution law.

i) Explain how variations in the parameter (a-b) with temperature might control where earthquake can nucleate. Which other parameter not directly contained in the RSF equations shown above is also important in controlling where a slip instability (i.e. an earthquake) might nucleate?

Question 5

a) Explain what is meant by dynamic recrystallization and outline the main mechanisms of microstructural change involved in this process.

b) How is the dynamically recrystallized grain size that develops during steady state dislocation-dominated flow typically related to flow stress and temperature? What is the theoretical basis for this relation?

c) Work by de Bresser et al has shown that dynamic recrystallization reorganises the grain size of a material to lie in the boundary between dislocation creep (grain size insensitive) and diffusion creep (grain size sensitive). Use this concept to obtain an alternative relationship between steady state flow stress and recrystallized grain size. Hint: Assume that dislocation creep is climb controlled with $3 \leq n \leq 5$ and diffusion creep is dominated by grain boundary diffusion.

d) Is the result you obtain similar to or different from that can be obtained using the Avrami equation (practical exercise done in class)? Why do you think the result is similar or different?

e) Explain under what circumstances dynamic recrystallization can lead to shear zone localization and whether or not recrystallized grain size versus stress relations can then be applied to estimate paleostresses.

Question 6

a) Explain what vacancies are and write down an equation for the equilibrium concentration of thermally produced vacancies in a pure elemental crystal maintained at a temperature T (K) and hydrostatic pressure P. Define all symbols appearing !!

b) Explain the statistical meaning of your answer to (a), with reference to the Boltzmann distribution law.

c) Derive an equation to show how the equilibrium concentration of vacancies is modified at a grain boundary transmitting a normal stress σ_n (superimposed on the hydrostatic component P), and hence explain the theoretical basis (driving force) for solid state diffusion creep.

d) Explain the possible pathways for vacancy and atomic diffusion during deformation by solid state diffusion creep, and identify the physically possible rate controlling steps using an electrical circuit analogue diagram.

e) At high temperatures (say 0.9 of the melting point in K), which transport path do you expect to be most important for vacancy diffusion in a fine grained material? Name the corresponding diffusion creep mechanism and write down the theoretical rate equation for this creep mechanism (no need to derive it!). Use the definition of enthalpy to show how the viscosity of a material deforming by this mechanism is expected to depend on temperature and pressure. Will flow at constant stress and temperature get easier or more difficult as pressure increases?

Question 7

a) Derive an expression for the flow strength of a crystal in the absence of linear defects and give an order of magnitude estimate for typical crystals.

b) Sketch what an edge dislocation might look like in a crystal and use your sketch to explain why the presence of linear defect helps to explain the flow strength of materials at high temperature.

c) Give the basic equation that describes the strain rate when deformation is controlled by the movement of dislocations. What is it called?

d) Use the equation in d) to derive a general flow law for materials deforming via climb-controlled creep.

e) What are typical conditions within the Earth under which your flow law derived in d) would apply for ***olivine***?